Star Formation

Star formation starts with the collapse of a cloud.

1. The Equilibrium of a Single Cloud

A cloud is subject to three forces: self-gravity, internal pressure (thermal, magnetic, turbulence, and rotation), and external pressure on its surface. Consider a spherical cloud in equilibrium under these forces. The cloud radius is R_c, mass M_c, and density ρ.

The mass in a spherical shell in this cloud is $\mathrm{d}M(r) = 4\pi r^2 \rho(r)\mathrm{d}r$. The inward force due to gravity is balanced by the outward force due to pressure drop across the shell: $4\pi r^2 \mathrm{d}P(r) = -GM(r)\mathrm{d}M(r)/r^2$. Rearranging this force equation, we get $3V(r)\mathrm{d}P(r) = -GM(r)\mathrm{d}M(r)/r$, where $V(r) = \frac{4}{3}\pi r^3$.

Integrate from the cloud center to the cloud surface. The integral of the right side is simply the gravitational energy Ω. The left side becomes:

$$3\int_{P_\mathrm{c0}}^{P_\mathrm{s}} V(r)\ \mathrm{d}P(r) \ = \ 3\ V(r)\ P(r)\ \Big|_\mathrm{center}^\mathrm{edge} - 3\int_0^{V_\mathrm{c}} P\ \mathrm{d}V \ = \ 3\ V_\mathrm{c}\ P_\mathrm{s} - 3\int_0^{V_\mathrm{c}} P\ \mathrm{d}V$$

where P_s is the pressure on the surface of the cloud, P_c0 is the pressure at the cloud center, and $V_\mathrm{c} \equiv \frac{4}{3}\pi R_\mathrm{c}^3$ is the cloud volume. For monatomic gas, the internal energy per unit volume is $\varepsilon_\mathrm{i} = \frac{3}{2}P$, and thus

$$3\int_0^{V_\mathrm{c}} P\ \mathrm{d}V \ = \ \frac{2}{3}\int_0^{V_\mathrm{c}} \varepsilon_\mathrm{i}\ \mathrm{d}V \ = \ \frac{2}{3}\ K$$

where K is the thermal energy content of the cloud. Therefore,

$$3\ V_\mathrm{c}\ P_\mathrm{s} \ = \ 2\ K \ + \ \Omega\ .$$

2. Spontaneous Star Formation - the Collapse of a Cloud

Assuming no surface pressure, $P_\mathrm{s} = 0$, and the equilibrium condition is $2\ K \ + \ \Omega = 0$, which is basically the Virial Theorem.

$2\ K \ + \ \Omega < 0$	cloud contracts
$= 0$	at equilibrium
> 0	cloud expands

For a cloud of uniform density ρ_c and pressure P_c,

$$K = \frac{3}{2} P_c V_c$$

and

$$\Omega = \int_0^{M_c} - \frac{G\, M(r)\, \mathrm{d}M}{r} = - \frac{16\pi^2}{3} \rho_c^2\, G \int_0^{R_c} r^4\, \mathrm{d}r = - \frac{3}{5} \frac{G\, M_c^2}{R_c} .$$

The condition for a cloud to contract is

$$\frac{3}{5} \frac{G\, M_c^2}{R_c} \geq 4\, \pi\, R_c^3\, P_c .$$

$P_c = n\, k\, T_c = \frac{\rho}{\mu\, m_H}\, k\, T_c$, where μ is the mean molecular weight, and m_H is the mass of a hydrogen atom. $M_c = \frac{4}{3}\, \pi\, R_c^3\, \rho_c$. Use these pressure and mass equations to eliminate the cloud mass or cloud radius in the condition for contraction, then we get

$$R_c \geq \left(\frac{15\, k\, T_c}{4\, \pi\, G\, \mu\, m_H\, \rho_c} \right)^{1/2} \equiv R_J \equiv \text{Jeans length}$$

and

$$M_c \geq \left(\frac{3}{4\, \pi\, \rho_c} \right)^{1/2} \left(\frac{5\, k\, T_c}{G\, \mu\, m_H} \right)^{3/2} \equiv M_J \equiv \text{Jeans mass} .$$

As a cloud collapses, if the heat generated is completely radiated away and the cloud temperature remains low, then the increasing cloud density lowers the local Jeans mass, so sub-regions of the cloud can satisfy the Jeans criterion and collapse independently. The cloud thus fragments to form multiple stars.

When the cloud core becomes optically thick and the heat can no longer be efficiently radiated away, the cloud temperature rises and so do the Jeans mass and Jeans length. The fragmentation of cloud stops.

3. Induced Star Formation

If a cloud is in a pressurized medium with a surface pressure of P_s, then the equilibrium condition would be

$$3\, V_c\, (\, P_c - P_s\,) = \frac{3}{5} \frac{G\, M_c^2}{R_c}$$

or

$$P_{\rm s} \;=\; P_{\rm c} - \frac{G\;M_{\rm c}^2}{5\;R_{\rm c}\;V_{\rm c}} \;=\; \left(\frac{3\;M_{\rm c}\;k\;T_{\rm c}}{4\pi\;\mu\;m_{\rm H}}\right)\frac{1}{R_{\rm c}^3} \;-\; \left(\frac{3\;G\;M_{\rm c}^2}{20\pi}\right)\frac{1}{R_{\rm c}^4} \;=\; \frac{A}{R_{\rm c}^3} \;-\; \frac{B}{R_{\rm c}^4}$$

where $A \;\equiv\; \frac{3\;M_{\rm c}\;k\;T_{\rm c}}{4\pi\;\mu\;m_{\rm H}}$ and $B \;\equiv\; \frac{3\;G\;M_{\rm c}^2}{20\pi}$. Figure 1 shows a sketch of equilibrium $P_{\rm s}$ as a function of $R_{\rm c}$. The maximum of $P_{\rm s}$ is at $R_{\rm c} \;=\; \frac{4B}{3A}$.

Suppose we start at an initial equilibrium surface pressure of P_0, then increase the pressure by ΔP. Two equilibrium cloud radii are possible, D and E. First let's consider D. After the external pressure increase, the cloud contracts a little bit, and when it reaches the radius D' equilibrium is reached again, so it is stable. Now we consider E. After the external pressure increase, the cloud has to expand in order to reach equilibrium. This is physically impossible. The cloud would contract as a response to the increased external pressure, and the contraction will continue. Therefore, clouds with radii smaller than $\frac{4B}{3A}$ are unstable and will collapse. Clouds with radii larger than $\frac{4B}{3A}$ are stable, but if the external pressure increases to $\geq \frac{3^3 A^4}{4^4 B^3}$ the cloud will be pushed to the unstable side on the left side of the plot in Figure 1 and collapse.

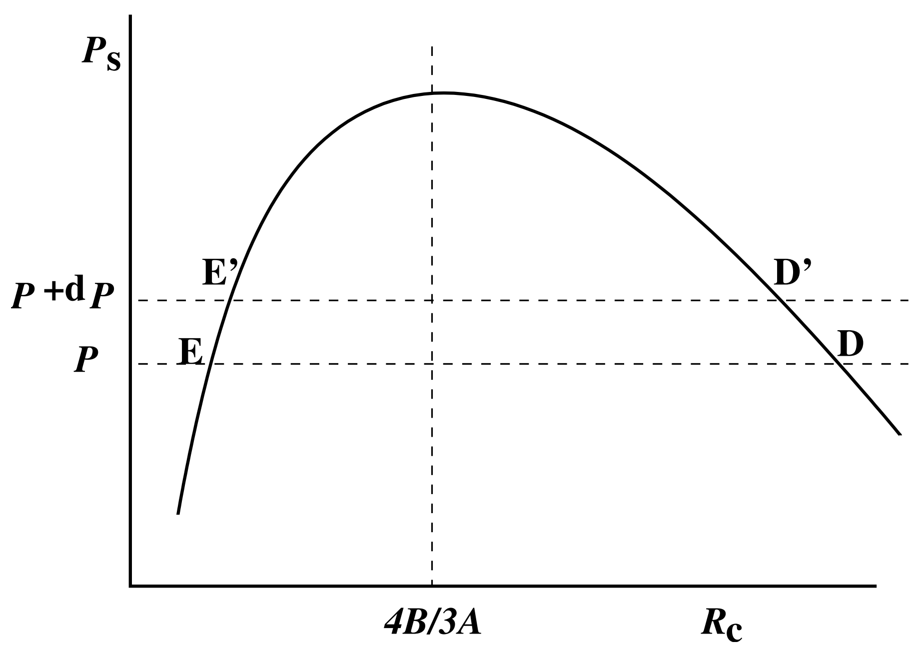

Figure 1. The pressure-radius relationship for an isothermal cloud with a surface pressure.