Photoionization and Recombination

At equilibrium:

\# of ionizations per unit volume
= # of recombinations per unit volume

$$N_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H^0)\, d\nu = N_e N_p \alpha(H^0, T)$$

J_ν : mean intensity of radiation, erg s^{-1} cm^{-2} Hz^{-1} Steradian^{-1}

N_{H^0} = Density of neutral H atom, cm^{-3}

a_ν = ionization cross section, cm^2

α = recombination coefficient, $cm^3\, s^{-1}$

$$a_\nu(Z) = \frac{A_0}{Z^2}\left(\frac{\nu_1}{\nu}\right)^4 \frac{e^{4-[(4\tan^{-1}\epsilon)/\epsilon]}}{1-e^{-2\pi/\epsilon}} \quad \text{for } \nu > \nu_1$$

where $A_0 = \frac{2^8\pi}{3e^4}\left(\frac{1}{137.0}\right)\pi a_0^2 = 6.30\times10^{-18}\ cm^2$

$$\epsilon = \sqrt{\frac{\nu}{\nu_1} - 1}$$

and $h\nu_1 = Z^2 h\nu_0 = 13.6\ Z^2\ eV$

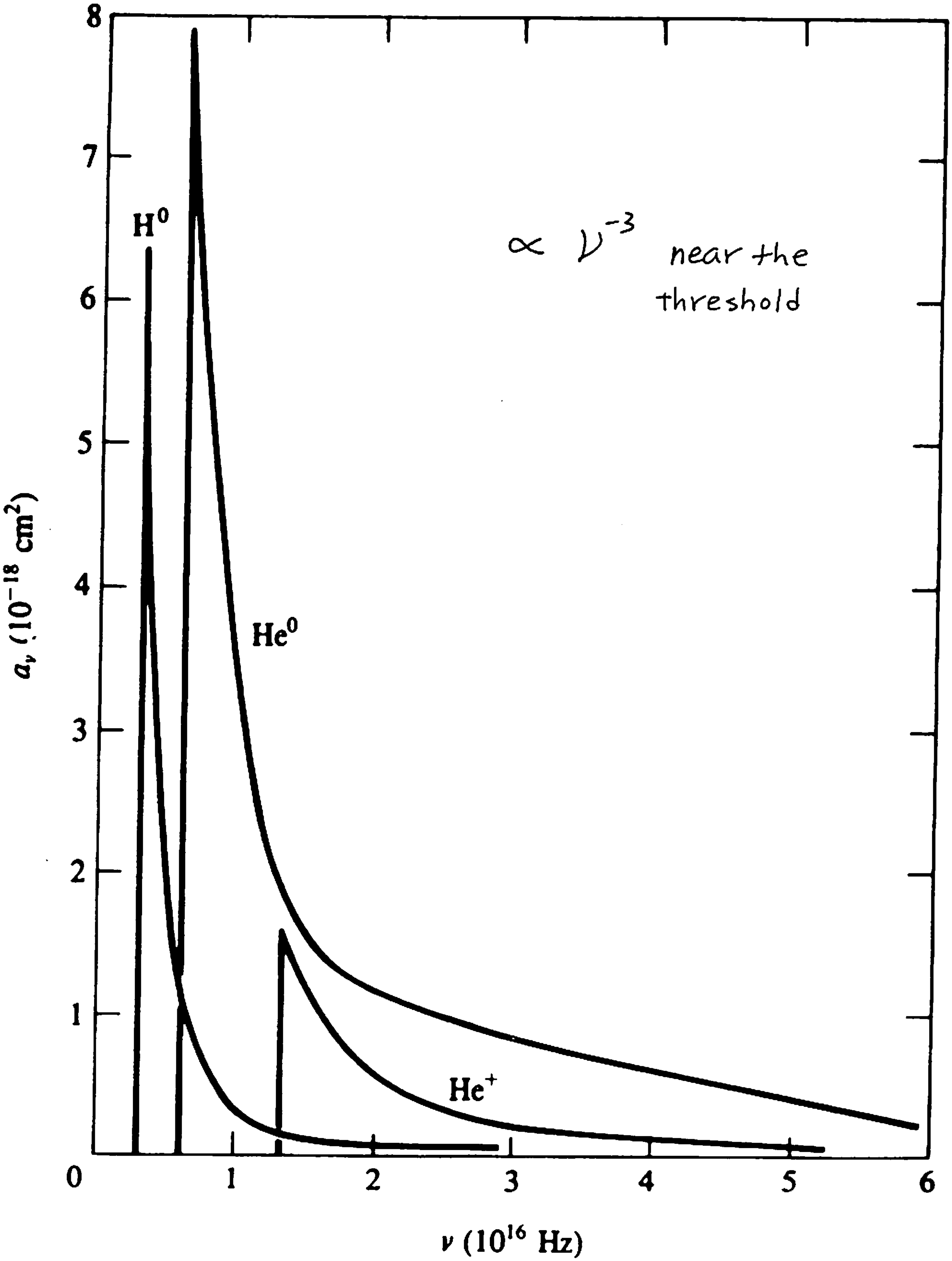


FIGURE 2.2
Photoionization absorption cross sections of H^0, He^0, and He^+.

Recombination coefficient to a specific level n^2L

$$\alpha_{n^2L}(H^\circ, T) = \int_0^\infty v\, \sigma_{nL}(H^\circ, v)\, f(v)\, dv ,$$

where $f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$

is the Maxwell-Boltzmann distribution function for the electrons.

$\sigma_{nL}(H^\circ, v)$ is the recombination cross section to the term n^2L for electrons with velocity v.

$$\sigma \propto v^{-2} \quad \Rightarrow \quad \alpha \propto T^{-1/2}$$

The total recombination coefficient

$$\alpha_A = \sum_{n,L} \alpha_{n^2L}(H^\circ, T) \qquad \text{(Table 2.1)}$$

A typical recombination time is $\frac{1}{N_e \alpha_A}$

$\alpha_A = 4.18 \times 10^{-13}\ \mathrm{cm^3\ sec^{-1}}$ at T 10^4 K

$\therefore$ recombination time is $\frac{7.6 \times 10^4}{N_e}$ yr

$$\boxed{\text{Approximately, recombination time} \approx \frac{10^5}{N_e}\ \text{yr.}}$$

What happens after a recombination?

How long does it take to photoionize an H°?

Recombination $\rightarrow$ H° at the excited level nL

$A_{nL,n'L'}$: transition probability from nL to $n'L'$
$\sim 10^4$ to 10^8 sec^{-1}

The mean lifetime of the excited level nL is

$$\tau_{nL} = \frac{1}{\sum_{n'<n} \sum_{L'=L\pm 1} A_{nL,n'L'}}$$

$\approx$ of order 10^{-4} to 10^{-8} sec.

H^0 at nL makes permitted one-photon downward transitions to the ground state 1^2S.

The 2^2S level is the only exception, for which two-photon emission is needed, and $A_{2^2S,1^2S} = 8.23\ sec^{-1}$. The life time of 2^2S is 0.12 sec.

$\boxed{\text{All cascade down to } 1^2S \text{ in} \ll 1 \text{ sec.}}$

At 5pc from an O6 star with an ionizing luminosity of $10^{48.7}$ photons/sec, the photoionization rate is $a(H^0)\frac{10^{48.7}}{4\pi d^2}$ sec^{-1} $\approx 10^{-8}$ sec^{-1}

$a(H^0) \approx 6\times10^{-18}$ cm^2

$d = 5\,pc = 1.5\times10^{19}$ cm

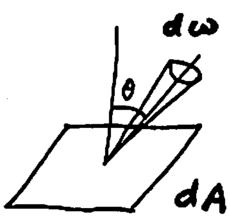
$\boxed{\text{The lifetime of } H^\circ \text{ against photoionization is } 10^8 \text{ sec} \sim 3 \text{ yr}}$

Photoionization of a Pure Hydrogen Nebula

$$N_{H^\circ} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu \, d\nu = N_p N_e \alpha_A (H^\circ, T)$$

J_ν : average intensity

I_ν : specific intensity erg cm^{-2} Hz^{-1} sec^{-1} steradian^{-1}



$$dE_\nu = I_\nu \, dt \, d\nu \, \mathbf{dA} \, \mathbf{d\omega}$$
$$= I_\nu \cos\theta \, dt \, d\nu \, dA \, d\omega$$

Radiative transfer (Spitzer p. 32-33)

$$\frac{dI_\nu}{ds} = - N_{H^\circ} a_\nu I_\nu + j_\nu ,$$

where j_ν is the local emission coefficient in erg cm^{-3} Hz^{-1} sec^{-1} steradian^{-1}

$$I_\nu = I_{\nu s} + I_{\nu d}$$

"stellar" "diffuse"

Stellar component:
$$4\pi J_{\nu s} = \frac{L_{\nu s}}{4\pi r^2} e^{-\int_0^r N_{H^\circ}(r') a_\nu dr'}$$

Diffuse component:
$$\frac{dI_{\nu d}}{ds} = - N_{H^\circ} a_\nu I_{\nu d} + j_\nu (T)$$

$$j_\nu (T) = \frac{2h\nu^3}{c^2} \left(\frac{h^2}{2\pi m k T} \right)^{3/2} a_\nu e^{-h(\nu-\nu_0)/kT} N_p N_e \qquad (\nu > \nu_0)$$

emitted by recombination to $1\,^2S$ level, strongly peaked to $\nu = \nu_0$, the threshold.

$J_{\nu s}$ and $J_{\nu d}$ can be calculated by an iterative procedure.

For an optically thin nebula,

$J_{\nu d} \approx 0$ is a good first approximation.

For an optically thick nebula,

every photon emitted by a recombination to 1^2S is immediately absorbed very close to the point where it is generated ("on the spot").

$$\boxed{N_{H^0} \int_{\nu_0}^{\infty} \frac{L_{\nu s}\, a_\nu}{4\pi r^2 h\nu} e^{-\int_0^r N_{H^0}(r')\, a_\nu\, dr'} d\nu = N_p N_e\, \alpha_B(H^0, T)}$$

where $\alpha_B(H^0, T) = \alpha_A(H^0, T) - \alpha_1(H^0, T)$

$L_{\nu s}$: luminosity of the star at ν

For any assumed density distribution

$$N_H(r) = N_{H^0}(r) + N_p(r)$$

and temperature distribution $T(r)$,

the above equation can be integrated outward to find $N_{H^0}(r)$ and $N_p(r) = N_e(r)$.

TABLE 2.2
Calculated ionization distributions for model H II regions

r(pc)	$T_* = 4 \times 10^4$ ° K Blackbody model		$T_* = 3.74 \times 10^4$ ° K Model stellar atmosphere	
	$\frac{N_p}{N_p + N_{H^0}}$	$\frac{N_{H^0}}{N_p + N_{H^0}}$	$\frac{N_p}{N_p + N_{H^0}}$	$\frac{N_{H^0}}{N_p + N_{H^0}}$
0.1	1.0	4.5×10^{-7}	1.0	4.5×10^{-7}
1.2	1.0	2.8×10^{-5}	1.0	2.9×10^{-5}
2.2	0.9999	1.0×10^{-4}	0.9999	1.0×10^{-4}
3.3	0.9997	2.5×10^{-4}	0.9997	2.5×10^{-4}
4.4	0.9995	4.4×10^{-4}	0.9994	4.5×10^{-4}
5.5	0.9992	8.0×10^{-4}	0.9992	8.1×10^{-4}
6.7	0.9985	1.5×10^{-3}	0.9985	1.5×10^{-3}
7.7	0.9973	2.7×10^{-3}	0.9973	2.7×10^{-3}
8.8	0.9921	7.9×10^{-3}	0.9924	7.6×10^{-3}
9.4	0.977	2.3×10^{-2}	0.979	2.1×10^{-2}
9.7	0.935	6.5×10^{-2}	0.940	6.0×10^{-2}
9.9	0.838	1.6×10^{-1}	0.842	1.6×10^{-1}
10.0	0.000	1.0	0.000	1.0

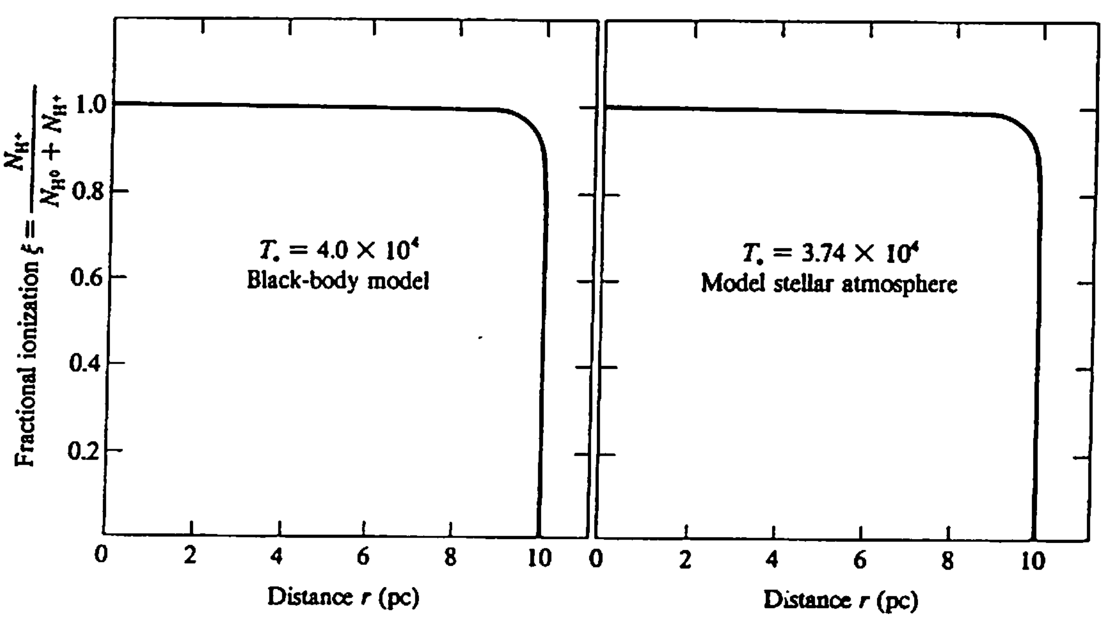


FIGURE 2.3
Ionization structure of two homogeneous pure-H model H II regions.

HII regions have sharp edges. The thickness of the edge is $d \approx \frac{1}{N_{H^\circ} a_\nu}$

for $N_{H^\circ}/N_H = 0.5$, $N_H = 10\ cm^{-3}$, $a_\nu = 6 \times 10^{-18}\ cm^2$

$d \approx 0.01$ pc

"Strömgren sphere"

For optically thick case,
total number of ionizing photons from the star
= total number of recombinations to excited levels

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\, d\nu = Q(H^\circ) = \frac{4}{3}\pi r_s^3 N_H^2 \alpha_B$$

$$r_s = \left(\frac{3\, Q(H^\circ)}{4\pi N_H^2 \alpha_B}\right)^{1/3}$$ radius of the Strömgren sphere

TABLE 2.3
Calculated radii of Strömgren spheres

Spectral type	M_v	$T_*(°K)$	Log $Q(H^0)$ (photons/sec)	Log $N_e N_p r_1^3$ (N in cm^{-3}; r_1 in pc)	r_1 (pc) ($N_e = N_p = 1\ cm^{-3}$)
O5	− 5.6	48,000	49.67	6.07	108
O6	− 5.5	40,000	49.23	5.63	74
O7	− 5.4	35,000	48.84	5.24	56
O8	− 5.2	33,500	48.60	5.00	51
O9	− 4.8	32,000	48.24	4.64	34
O9.5	− 4.6	31,000	47.95	4.35	29
B0	− 4.4	30,000	47.67	4.07	23
B0.5	− 4.2	26,200	46.83	3.23	12

NOTE: $T = 7500°$ K assumed for calculating α_B.

Photoionization of a Nebula Containing H and He

ionization potential	H°	13.6 eV	$\equiv h\nu_0$
	He°	24.6 eV	$\equiv h\nu_2$
	He^{+}	54.4 eV	

13.6eV $< h\nu <$ 24.6eV can ionize only H°
24.6eV $< h\nu$ can ionize both H°, He°

$\therefore$ The ionization equations of H and He are coupled by the radiation field with $h\nu > 24.6$ eV.

Considerations in calculations of H + He nebula:

1. Photons emitted by recombinations to the ground level of He can ionize either H or He.
2. Photons emitted by recombinations to excited levels of He can ionize only H.
3. Recombinations to triplets will cascade down to 2^3S.
   (i) $2^3S \rightarrow 1^1S$ emits one photon 19.8 eV
   $A_{2^3S,1^1S} = 1.27 \times 10^{-4}$ sec^{-1} (lifetime ~ 2^{hr})
   (ii) collisional excitation to 2^1S or 2^1P

$$Ne\, q_{2^3S,\,2^1L} = Ne \int_{\frac{1}{2}mv^2=\chi}^{\infty} v\, \sigma_{2^3S,2^1L}(v)\, f(v)\, dv$$

where χ is the energy threshold
$\sigma_{2^3S,2^1L}$ is the electron collision cross section

when $Ne \lesssim 10^2$, one-photon emission dominates
$Ne \approx 10^4$, collisional excitation is important

4. Recombinations to singlet-excited levels will cascade down to 2^1S and 2^1P

$2^1P \rightarrow 1^1S$ emits $h\nu = 21.2$ eV, resonance line

$2^1P \rightarrow 2^1S$ emits $2.06\,\mu$ photon

$$\frac{A_{2^1P,\,1^1S}}{A_{2^1P,\,2^1S}} \sim 10^3$$

Resonance-line photons are scattered by He^0; most are absorbed by H^0, but some end up in a $2.06\,\mu$ photon and a 2^1S

$2^1S \rightarrow 1^1S$ by two-photon emission, $\sim 56\%$ of the photons can ionize H^0.

In the on-the-spot approximation, the ionization equations become

Eq (1) →
$$N_{H^\circ} \int_{\nu_0}^{\infty} \frac{L_{\nu s}}{4\pi r^2 h\nu} a_\nu(H^\circ) e^{-\tau_\nu} d\nu + y\, N_{He^+} N_e\, \alpha_1(He^\circ, T) + p\, N_{He^+} N_e\, \alpha_B(He^\circ, T) = N_p N_e\, \alpha_B(H^\circ, T)$$

Eq (2) →
$$N_{He^\circ} \int_{\nu_2}^{\infty} \frac{L_{\nu s}}{4\pi r^2 h\nu} a_\nu(He^\circ) e^{-\tau_\nu} d\nu + (1-y) N_{He^+} N_e\, \alpha_1(He^\circ, T) = N_{He^+} N_e\, \alpha_A(He^\circ, T)$$

with
$$\frac{d\tau_\nu}{dr} = N_{H^\circ} a_\nu(H^\circ) \quad \text{for } \nu_0 < \nu < \nu_2$$
$$N_{H^\circ} a_\nu(H^\circ) + N_{He^\circ} a_\nu(He^\circ) \quad \text{for } \nu_2 < \nu$$

Eq (3) →
$$N_e = N_p + N_{He^+}$$

These equations can be integrated outward step-by-step

Good approximation
$$\int_{\nu_2}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(He^\circ) = \frac{4\pi}{3} r_2^3 N_{He^+} N_e\, \alpha_B(He^\circ)$$
$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(H^\circ) = \frac{4\pi}{3} r_1^3 N_{H^+} N_e\, \alpha_B(H^\circ)$$
$$\left(\frac{r_1}{r_2}\right)^3 = \frac{Q(H^\circ)}{Q(He^\circ)} \frac{N_{He}}{N_H} \left(\frac{N_H + N_{He}}{N_H}\right) \frac{\alpha_B(He^\circ)}{\alpha_B(H^\circ)}$$

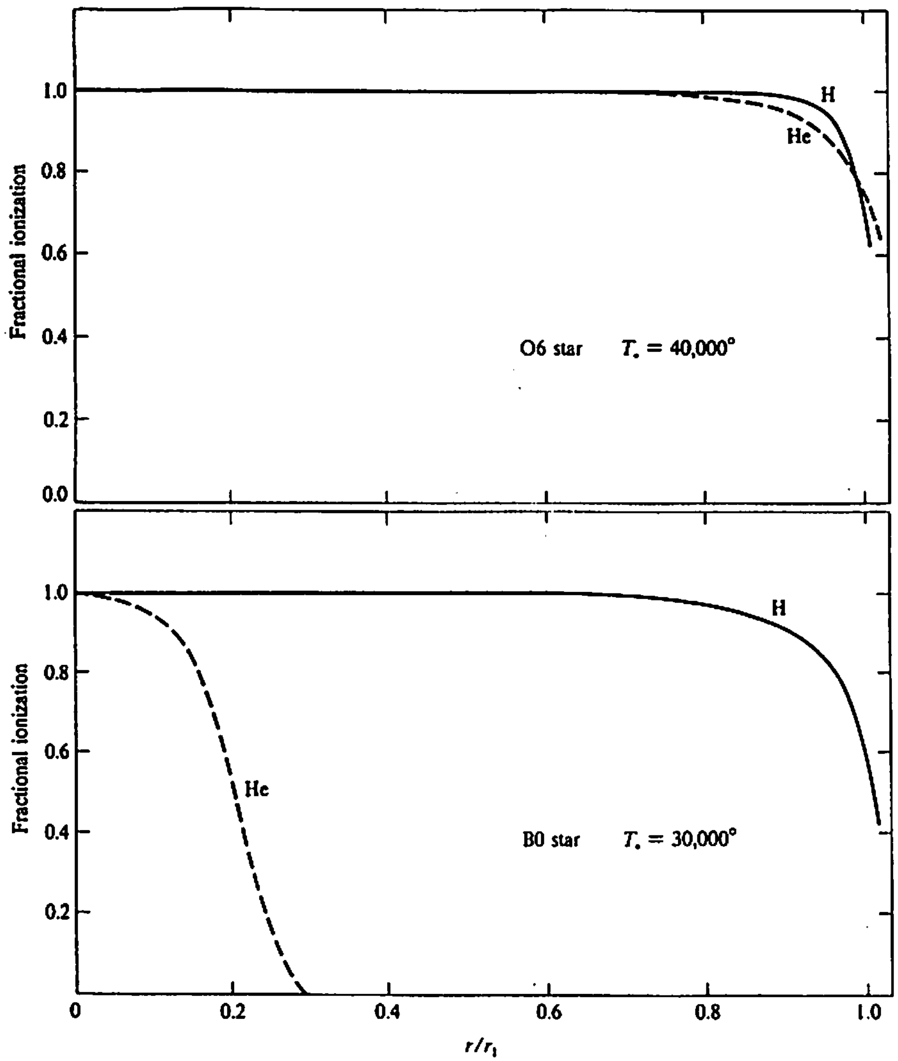


FIGURE 2.4
Ionization structure of two homogeneous H + He model H II regions.

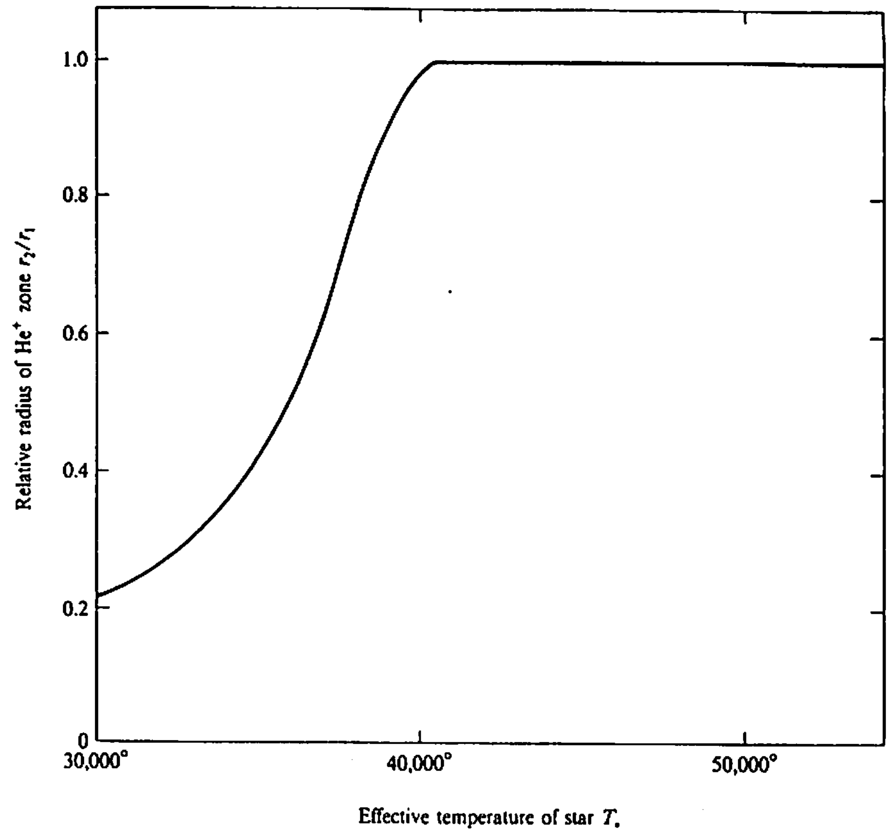


FIGURE 2.5
Relative radius of He^+ zone as a function of effective temperature of exciting star.

Photoionization of He^+ to He^{++}

$He^+ \to He^{++}$ ionization potential 54.4 eV

Normal O stars do not emit much at $h\nu > 54.4$ eV.
Some extreme WR stars, PN nuclei, x-ray binaries do emit photons with $h\nu > 54.4$ eV.

Keep track of He^{++} recombinations, emitted radiation can ionize H^0, ...

Strömgren radius r_3

$$\int_{4\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = \frac{4}{3}\pi r_3^3 N_{He^{++}} N_e \alpha_B(He^+, T)$$

For stellar temperature $T_* \gtrsim 10^5$ K

$$\frac{r_3}{r_1} \sim 1.$$

PN nuclei can reach such high temperatures.

He II $\lambda 4686$ emission is frequently seen in PNe but not in HII regions.

He II - emitting HII regions are very rare.

Example: N44C, LMC X-1 nebula, Br2 nebula (WN2), etc.

Heavy Elements Included

When heavy elements are included in the photo-ionization model, the following processes need to be considered:

* Charge exchange reaction

  ionization potential of H° 13.598 eV

  ionization potential of O° 13.618 eV

$$O^0(^3P) + H^+ \rightarrow O^+(^4S) + H^\circ(^2S)$$

* Dielectronic charge exchange

$$O^{+2}(2s^2 2p^2\ ^3P) + H^0(1s\ ^2S) \rightarrow O^+(2s\ 2p^4\ ^4P) + H^+$$

At 10^4K, $O^{+2} \rightarrow O^+$

(α_R) radiative recombination coef. 5.43×10^{-12} cm³/sec

(δ') charge exchange reaction coef. 0.77×10^{-9} cm³/sec

(α_d) dielectronic recombination coef. 1.14×10^{-11} cm³/sec

As T $\uparrow$ $\alpha_R \downarrow$

$\delta' \uparrow$

$\alpha_d \downarrow$

At high temperatures, charge-exchange reactions become even more important.