Photoionization and Recombination

At equilibrium:

# of ionizations per unit volume
= # of recombinations per unit volume

$$N_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H^0)\, d\nu = N_e N_p \alpha(H^0, T)$$

J_ν : mean intensity of radiation, $erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ steradian^{-1}$

N_{H^0} = No. of neutral H atom, cm^{-3}

a_ν = ionization cross section, cm^2

α = recombination coefficient, $cm^3\ s^{-1}$

$$a_\nu(Z) = \frac{A_0}{Z^2}\left(\frac{\nu_1}{\nu}\right)^4 \frac{e^{4-[(4\tan^{-1}\epsilon)/\epsilon]}}{1-e^{-2\pi/\epsilon}} \quad \text{for } \nu > \nu_1$$

where $A_0 = \frac{2^8\pi}{3e^4}\left(\frac{1}{137.0}\right)\pi a_0^2 = 6.30\times10^{-18}\ cm^2$

$$\epsilon = \sqrt{\frac{\nu}{\nu_1} - 1}$$

and $h\nu_1 = Z^2 h\nu_0 = 13.6\ Z^2\ eV$

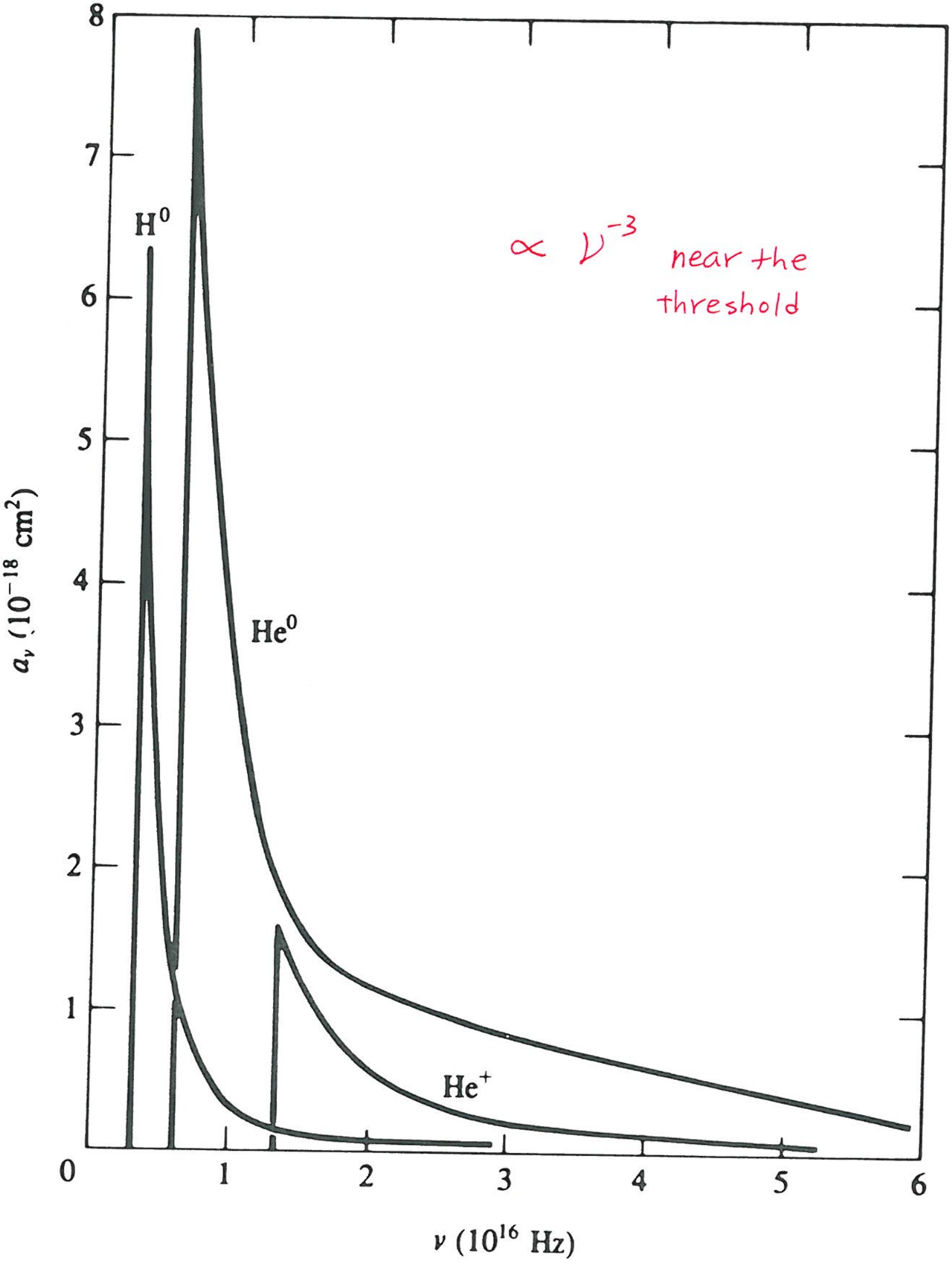

FIGURE 2.2
Photoionization absorption cross sections of H^0, He^0, and He^+.

Recombination coefficient to a specific level n^2L

$$\alpha_{n^2L}(H^0, T) = \int_0^\infty v\, \sigma_{nL}(H^0, v)\, f(v)\, dv,$$

where $f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$

is the Maxwell-Boltzmann distribution function for the electrons.

$\sigma_{nL}(H^0, v)$ is the recombination cross section to the term n^2L for electrons with velocity v.

$$\sigma \propto v^{-2} \quad \Rightarrow \quad \alpha \propto T^{-1/2}$$

The total recombination coefficient

$$\alpha_A = \sum_{n,L} \alpha_{n^2L}(H^0, T) \qquad \text{(Table 2.1)}$$

A typical recombination time is $\frac{1}{N_e \alpha_A}$

$\alpha_A = 4.18 \times 10^{-13}\ \text{cm}^3\ \text{sec}^{-1}$ at 10^4 K

$\therefore$ recombination time is $\frac{7.6 \times 10^4}{N_e}$ yr

Approximately, recombination time $\approx \frac{10^5}{N_e}$ yr.

What happens after a recombination?

How long does it take to photoionize an H^0?

Recombination → H° at the excited level nL

$A_{nL,n'L'}$: transition probability from nL to $n'L'$
$\sim 10^4$ to 10^8 sec^{-1}

The mean lifetime of the excited level nL is

$$\tau_{nL} = \frac{1}{\sum_{n'<n} \sum_{L'=L\pm1} A_{nL,n'L'}}$$

$\approx$ of order 10^{-4} to 10^{-8} sec.

H° at nL makes permitted one-photon downward transitions to the ground state 1^2S.

The 2^2S level is the only exception, for which two-photon emission is needed, and $A_{2^2S,1^2S} = 8.23\ sec^{-1}$.
The life time of 2^2S is 0.12 sec.

All cascade down to 1^2S in $\ll 1$ sec.

At 5pc from an O6 star with an ionizing luminosity of $10^{48.7}$ photons/sec, the photoionization rate is $a(H^\circ)\frac{10^{48.7}}{4\pi d^2}\ sec^{-1} \approx 10^{-8}\ sec^{-1}$

$a(H^\circ) \approx 6\times10^{-18}\ cm^2$

$d = 5\,pc = 1.5\times10^{19}\ cm$

The lifetime of H° against photoionization is 10^8 sec $\sim$ 3 yr