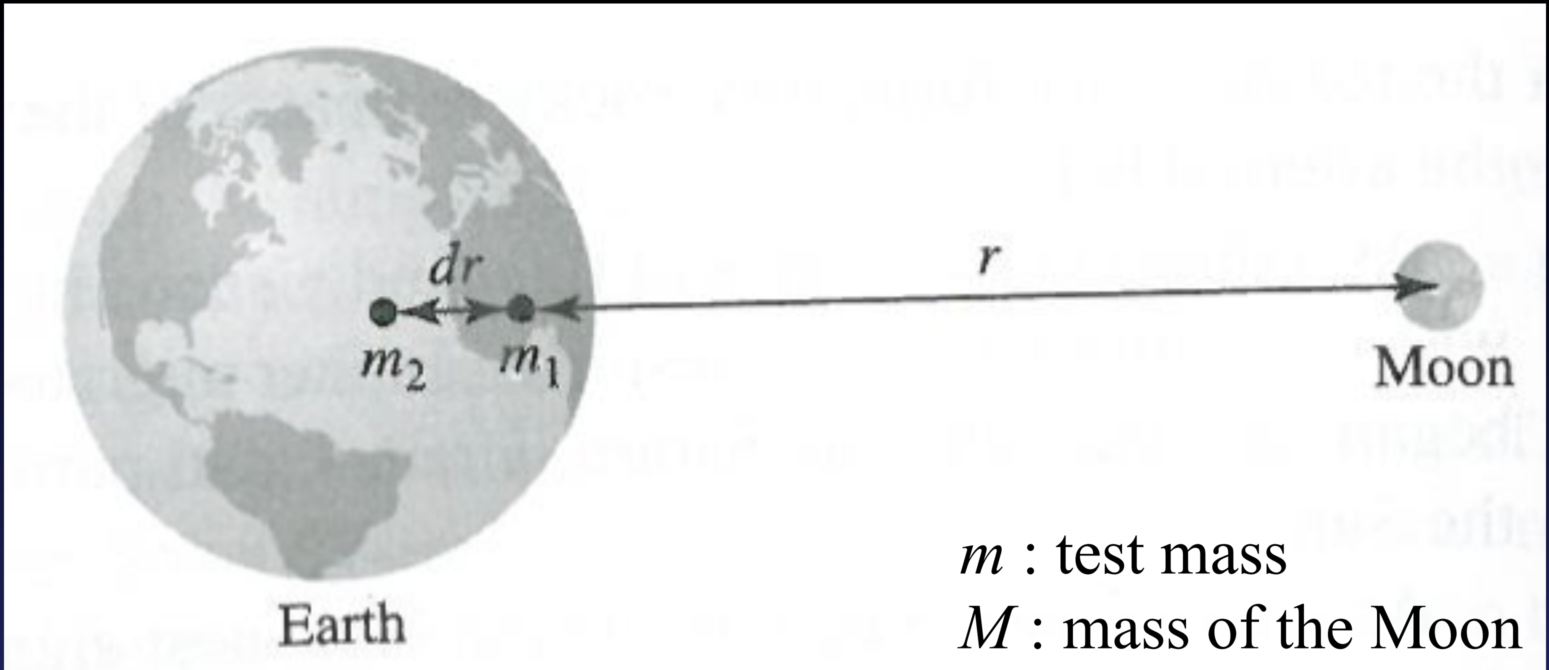
Astronomy 405
Solar System and ISM

Lecture 3:
Tidal Forces

January 18, 2013

Tides -- twice a day

The Bay of Fundy at low and high tides

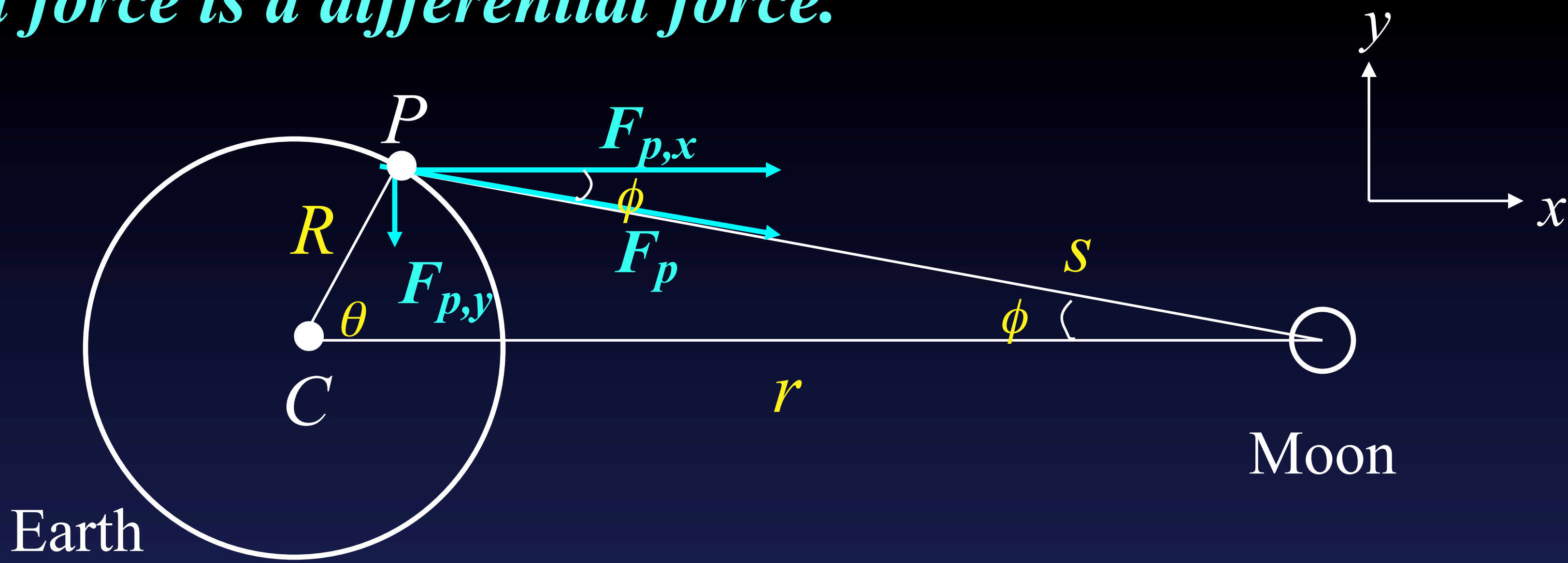


Gravitational Force $$F_m = G \frac{M\,m}{r^2}$$

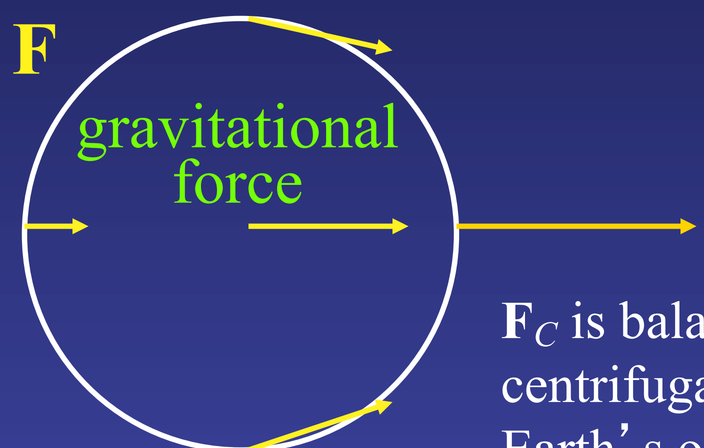
Tidal force is a differential force:

$$d\,F_m = \left(\frac{d\,F_m}{d\,r}\right) dr = -\,2G\,\frac{M\,m}{r^3}\,dr$$

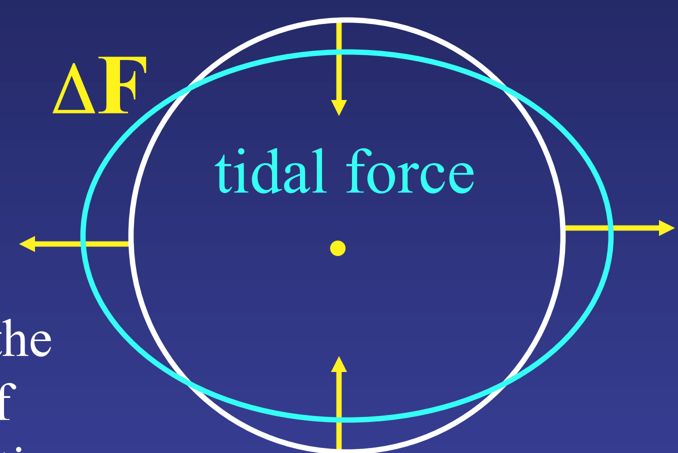
Tidal force is a differential force.



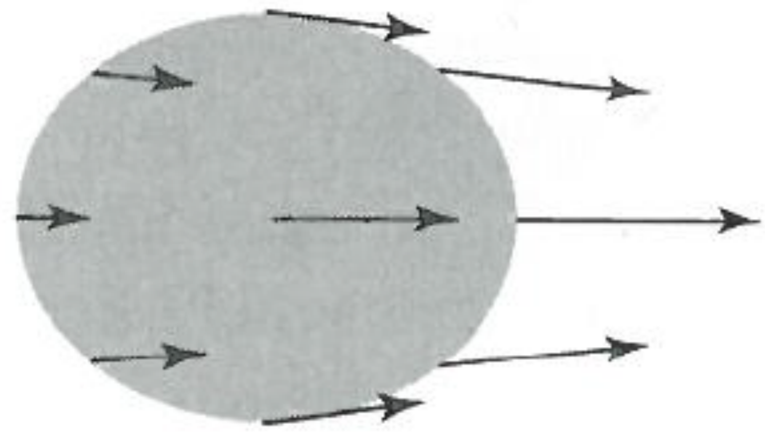
$$\Delta \mathbf{F} = \mathbf{F}_P - \mathbf{F}_C \approx \frac{G\,M\,m\,R}{r^3}\left(2\cos\theta\ \hat{\mathbf{i}} - \sin\theta\ \hat{\mathbf{j}}\right)$$



$\mathbf{F}_C$ is balanced by the centrifugal force of Earth's orbital motion

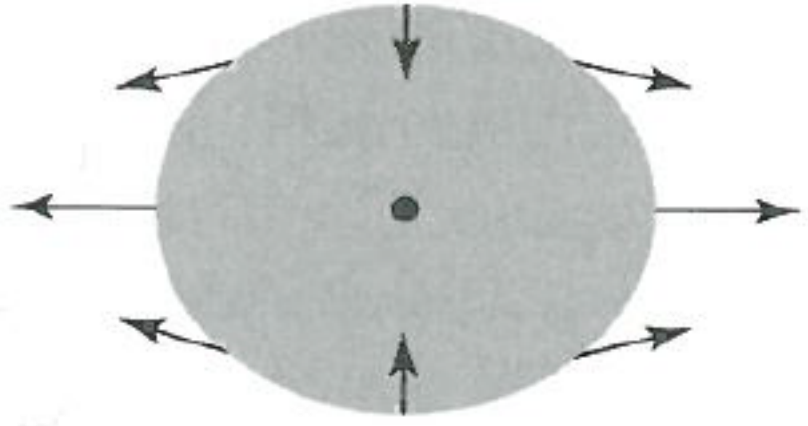


Gravitational Force =>



(a)

Differential Gravitational Force => (Tidal force)



Tidal force $\propto$ M R / r^3

Tidal force on Moon due to Earth : Tidal force on Earth due to Moon

$$= (M_{earth} R_{moon}) / (M_{moon} R_{earth})$$

$$= (M_{earth} / M_{moon}) (R_{moon} / R_{earth})$$

$$= (6 \times 10^{24} / 7.3 \times 10^{22}) (1.7 \times 10^{6} / 6.4 \times 10^{6})$$

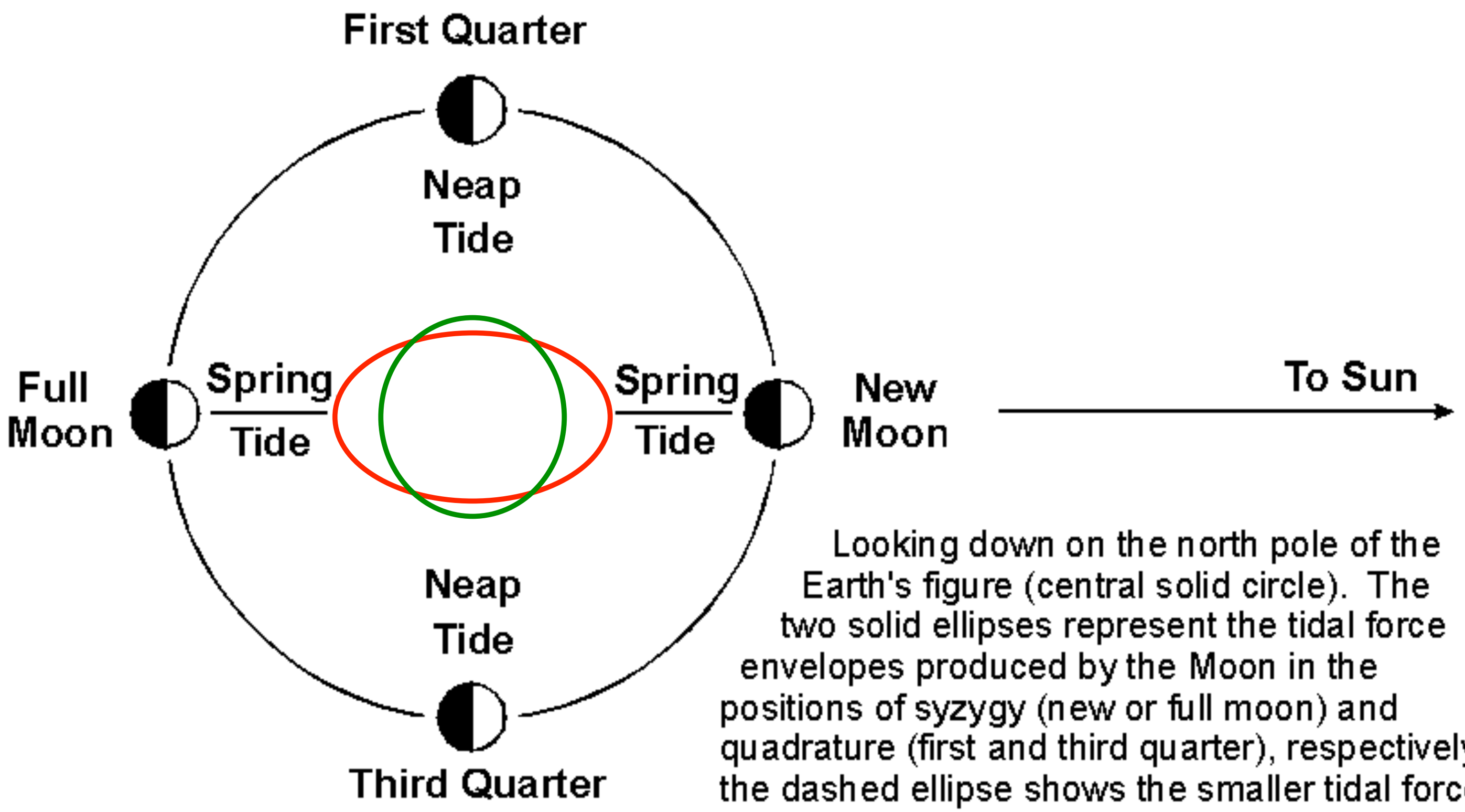
$$= 22$$

Tidal bulge of the solid Earth ~ 10 cm in height
Tidal bulge of the Moon ~ 20 m

Both Moon and Sun exert tides on Earth.

$M_{sun}/M_{moon} \sim 3\times10^7$, $D_{sun}/D_{moon} \sim 390$

$$\Delta F_{sun}/\Delta F_{moon} \sim (3\times10^7) \times (390)^{-3} \sim 0.5$$



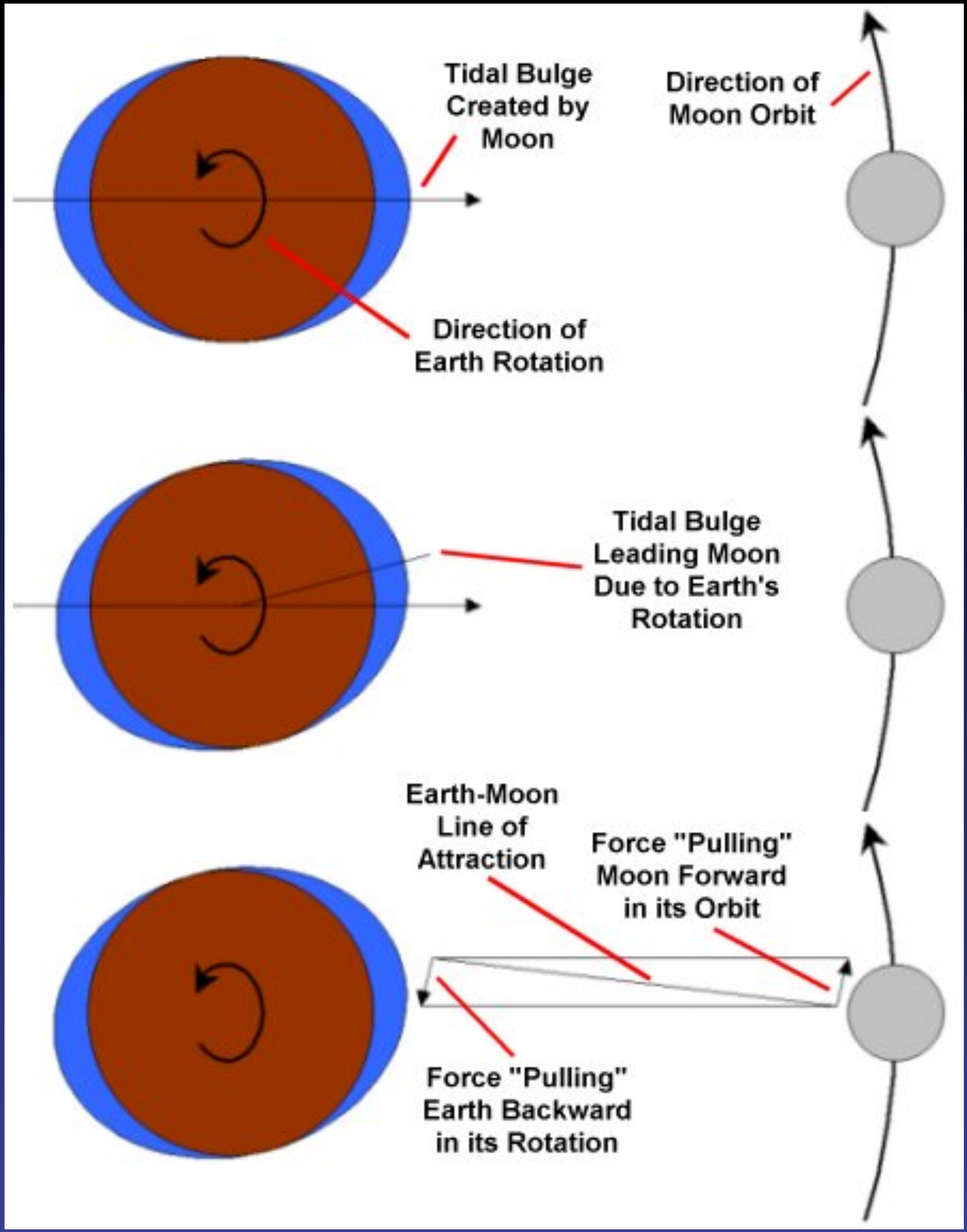
Looking down on the north pole of the Earth's figure (central solid circle). The two solid ellipses represent the tidal force envelopes produced by the Moon in the positions of syzygy (new or full moon) and quadrature (first and third quarter), respectively; the dashed ellipse shows the smaller tidal force envelope produced by the Sun.

The Earth rorates faster than the moon orbits.

Friction is dissipative, so rotational kinetic energy is constantly lost.

Earth’s rotational period is lengthening at a rate of 0.0016 s /century

The Moon gains speed and drifts away at a rate of 3-4 cm /yr.



The Earth's tidal bulge leads the Moon, so the Earth's rotation is slowed down gradually. Eventually it will have a rotational period equal to the orbital period - synchronous rotation.

The Earth exerts much stronger tidal forces on the Moon. The Moon has reached synchronous rotation. We always see the same side of the Moon.

Synchronous rotation is seen in:
Phobos and Deimos of Mars
Galilean moons of Jupiter
Charon and Pluto

Synchronous orbit
Moon's orbital period = planet' s rotation period

Planet's tidal bulge leads the moon's orbit => gain speed,
spiral outward

Planet' s tidal bulge trails the moon's orbit => lose speed
spiral in

Inside synchronous orbit,
spiral in no matter pro- or
retrograde orbits.

Outside sychronous orbit,
Prograde => spiral out
Retrograde => spiral in

Roche limit:
when tidal force is greater than the gravitational force that holds the body together

$$\frac{G M_m}{R_m^{\,2}} < \frac{2 G M_p R_m}{r^3}$$

m : moon, p : planet

r : distance

$$r < f_R \, (\rho_p / \rho_m)^{1/3} R_p$$

where $f_R = 2^{1/3} = 1.3$

Roche Limit
$f_R = 2.456$

Saturn density = 0.71 g cm^{-3}

Moon density = 1.2 g cm^{-3}

Saturn radius = 6 x 10^{9} cm

r = 1.24 x 10^{10} cm

All rings are within the Roche limit.