Chapter 8

Star formation and star forming regions

8.1 Introduction

The average density in a star is in excess of 10^3 kg m^{-3}, and this is enormously greater than any densities encountered in the interstellar gas. Evidently, great compression must occur, and the only force capable of producing this in a mass of gas is the self-gravity of the gas. However, gravity must overcome a variety of disruptive forces. For example, gas and magnetic pressure, turbulence and rotation, all act against compression. Indeed, so many are the effects opposing star formation that the difficulties seem almost insuperable. However, it is obvious from the structure of the Galaxy that Nature has no such difficulties!

The existence of stars which have lifetimes much less than the age of the Galaxy implies that star formation must be an ongoing process. This is further suggested by heavy element abundances which clearly show that many cycles of nuclear burning must have occurred. We will consider only the simplest possible situations involving star formation and look at necessary—but not sufficient (!)—conditions which must be satisfied for star formation to take place.

8.1.1 The equilibrium of a single cloud

We first consider a single isolated spherically symmetric cloud in equilibrium under three forces, namely internal pressure, self-gravity and surface pressure exerted by an external medium. To establish the necessary equations, consider a spherical shell in the cloud of thickness dr at radius r (figure 8.1). The shell has mass

$$\mathrm{d}M(r) = 4\pi r^2 \rho(r)\,\mathrm{d}r \qquad (8.1)$$

where $\rho(r)$ is the density of the gas at radius r. In equilibrium, the inwards gravitational force on the shell due to the mass $M(r)$ interior to it must be balanced by a pressure gradient. Thus a pressure differential d$P(r)$ must exist across the shell. Obviously, the pressure must decrease outwards to produce an

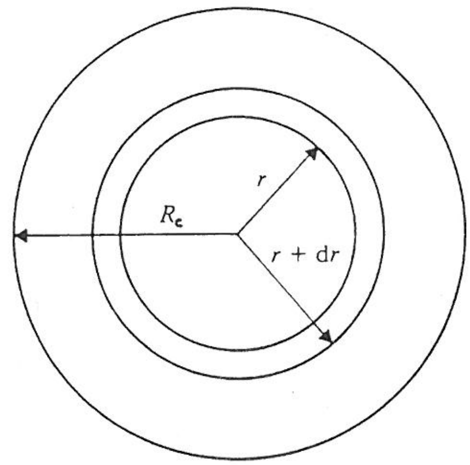

Figure 8.1. Geometry of a spherical shell.

outwards force. For equilibrium, therefore,

$$4\pi r^2 \,\mathrm{d}P(r) = -GM(r)\,\mathrm{d}M(r)/r^2. \tag{8.2}$$

We can write this equation as

$$3V(r)\,\mathrm{d}P(r) = -GM(r)\,\mathrm{d}M(r)/r \tag{8.3}$$

where $V(r) \equiv \frac{4}{3}\pi r^3$ is the interior volume at radius r. Let us now integrate equation (8.3) from the centre of the cloud to its edge where it has radius R_c and pressure equal to the external pressure P_s. Thus

$$3\int_{P_{c0}}^{P_s} V(r)\,\mathrm{d}P(r) = -\int_0^{M_c} \frac{GM(r)\,\mathrm{d}M}{r}. \tag{8.4}$$

In equation (8.4), M_c is the total mass of the cloud and P_{c0} is the central pressure in the cloud. The left-hand side of equation (8.4) can be integrated by parts and gives

$$\begin{aligned} 3\int_{P_{c0}}^{P_s} V(r)\,\mathrm{d}P(r) &= 3V(r)P(r)\Big|_{\text{centre}}^{\text{edge}} - 3\int_0^{V_c} P\,\mathrm{d}V \\ &= 3V_c P_s - 3\int_0^{V_c} P\,\mathrm{d}V. \end{aligned} \tag{8.5}$$

Here $V_c \equiv \frac{4}{3}\pi R_c^3$ is the volume of the cloud.

Since the internal energy, ε_i, per unit volume of monatomic gas is given by

$$\varepsilon_i = \frac{3}{2}P \tag{8.6}$$

we can write

$$\int_0^{V_c} P\,\mathrm{d}V = \frac{2}{3}\int_0^{V_c} \varepsilon_I\,\mathrm{d}V = \frac{2}{3}\mathcal{T} \tag{8.7}$$

where $\mathcal{T}$ is the thermal energy content of the cloud.

The right-hand side of equation (8.4) is just the gravitational self-energy, Ω, of the cloud. Equation (8.4) can be written as

$$3V_c P_s = 2\mathcal{T} + \Omega. \tag{8.8}$$

Other forces (e.g. magnetic fields) could be added to this formulation.

8.1.2 The collapse of an isolated gas cloud and spontaneous star formation

We will initially consider a spherical gas cloud on which the only forces acting are those due to its self-gravity and its internal pressure. We first ignore the surface pressure exerted by any surrounding gas, since the conclusions are not drastically changed by its inclusion, but discuss later its significance (section 8.1.3). We have already derived a criterion which must hold if the cloud is to be in equilibrium. Applied to these circumstances it takes the form of equation (8.8) with the surface pressure term removed, i.e.

$$2\mathcal{T} + \Omega = 0. \tag{8.9}$$

Suppose we have a situation where $2\mathcal{T} > -\Omega$, i.e. $2\mathcal{T} + \Omega > 0$. Then the pressure term dominates and intuitively we would expect the cloud to expand. Conversely, if $2\mathcal{T} < -\Omega$, i.e. $2\mathcal{T} + \Omega < 0$, we would expect the cloud to be contracting. These intuitive expectations can be confirmed by analysis and we can write the following schematic equation to describe the dynamical state of an isolated cloud:

$$\begin{aligned} 2\mathcal{T} + \Omega &< 0 \quad \text{Contraction} \\ &= 0 \quad \text{Equilibrium} \\ &> 0 \quad \text{Expansion.} \end{aligned} \tag{8.10}$$

Equation (8.3) shows that the pressure and density in the cloud will vary with radius. This variation can be found by integrating this equation subject to assumptions connecting the cloud pressure and density. In order to make simple estimates we will assume that the cloud has a uniform density ρ_c and pressure P_c. For a uniform cloud then

$$2\mathcal{T} = 3P_c V_c \tag{8.11}$$

and

$$\Omega = -\int_0^{M_c} \frac{GM(r)\,\mathrm{d}M}{r} = -\frac{16\pi^2}{3}\rho_c^2 G \int_0^{R_c} r^4\,\mathrm{d}r = -\frac{3}{5}\frac{GM_c^2}{R_c}. \tag{8.12}$$

Thus, in order to start the contraction which is necessary if a cloud is ultimately to form a star (or stars) we require

$$\frac{3}{5}\frac{GM_c^2}{R_c} \gtrsim 4\pi R_c^3 P_c. \tag{8.13}$$

Since

$$P_c = \frac{3M_c kT_c}{4\pi R_c^3 m_H \mu} \tag{8.14}$$

where T_c is the gas temperature (assumed constant) condition (8.13) becomes

$$\frac{GM_c}{5R_c} \gtrsim \frac{kT_c}{\mu m_H}. \tag{8.15}$$

Now, $kT_c/\mu m_H = c_c^2$, where c_c is the sound speed in the cloud (which will be assumed to behave isothermally). The sound travel time across the cloud is $t_s \approx R_c/c_c$. Hence equation (8.15) can be written as

$$t_s \gtrsim (15/4\pi G\rho_c)^{1/2}. \tag{8.16}$$

The term on the right-hand side of condition (8.16) has the following physical interpretation. Consider a spherical cloud which is allowed to collapse under self-gravity. If we ignore the effects of the internal pressure then this is said to be a free-fall collapse. The equation of motion of a thin shell situated an initial distance r_0 from the centre (figure 8.1) is just

$$\frac{d^2 r}{dt^2} = -\frac{4}{3}\frac{\pi G r_0^3 \rho_c}{r^2}. \tag{8.17}$$

In deriving this equation, we have assumed that the cloud has an initially uniform density ρ_c. Define the following quantities:

$$x = r/r_0, \qquad \tau = t/t_{ff}, \qquad t_{ff} = \sqrt{3\pi/32G\rho_c}. \tag{8.18}$$

Equation (8.17) then takes the form

$$\ddot{x} = -\frac{\pi^2}{8x^2} \tag{8.19}$$

where the dot indicates differentiation with respect to τ. We can put equation (8.19) into the form

$$\frac{d}{dx}\dot{x}^2 = -\frac{\pi^2}{4x^2} \tag{8.20}$$

and integrate to get

$$\dot{x}^2 = \frac{\pi^2}{4x} + a. \tag{8.21}$$

Here a is a constant of integration. If the cloud starts off from rest, then when $x = 1$, $\dot{x} = 0$ and equation (8.21) becomes

$$\dot{x} = -\frac{\pi}{2}\left(\frac{1}{x} - 1\right)^{1/2}. \tag{8.22}$$

(Note that we take the negative root since $\dot{x}$ is directed inwards.) In order to integrate this equation, set $x = \cos^2\theta$. Then equation (8.22) becomes

$$\cos^2\theta\,\dot{\theta} = \frac{\pi}{4} \tag{8.23}$$

which can be immediately integrated to give

$$\frac{\theta}{2} + \frac{1}{4}\sin 2\theta = \frac{\pi\tau}{4} + b \tag{8.24}$$

where b is the constant of integration. When $\tau = 0$, $x = 1$ so that $\theta = 0$. Hence $b = 0$. Now the shell reaches the centre when $x = 0$, i.e. when $\theta = \pi/2$. From equation (8.24) we thus obtain $\tau = 1$ when $x = 0$. Note that this time is the same for all r_0. Hence the free-fall collapse time is just

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho_{\rm c}}\right)^{1/2}. \tag{8.25}$$

We now see that the condition for collapse (equation (8.16)) can be written in the form

$$t_{\rm s} \gtrsim \frac{2\sqrt{10}}{\pi} t_{\rm ff} \approx 2t_{\rm ff}. \tag{8.26}$$

Physically then the condition for collapse is that the free-fall time must be less than about the time taken for a sound wave to cross the cloud. We can write equation (8.26) in an alternative form by defining $M_{\rm crit}$ as the critical mass for collapse of a cloud of density $\rho_{\rm c}$ and internal sound speed $c_{\rm c}$. This critical mass is often referred to as the Jeans mass. Then equation (8.26) gives as the collapse condition

$$M \gtrsim M_{\rm crit} = \left(\frac{3\pi^5}{32}\right)^{1/2} c_{\rm c}^3 G^{-3/2} \rho_{\rm c}^{-1/2}. \tag{8.27}$$

Let us now calculate the critical mass under the density and temperature conditions typical of the diffuse neutral clouds discussed in section 7.4.1. We find $M_{\rm crit} \approx 10^4 M_\odot$. This is much greater than their observed masses. We conclude that star formation does not take place in such clouds and ample evidence confirms this conclusion.

If we apply these ideas to the gas in cool molecular clouds, we come to rather different conclusions. We will assume that the typical particle density is $5 \times 10^9\ \mathrm{m^{-3}}$ and the gas temperature is 20 K. The critical mass then is $M_{\rm crit} \approx 30 M_\odot$, which is derived assuming that the mean mass of a particle is $2m_{\rm H}$. The critical radius corresponding to this mass is $R_{\rm crit} \approx 0.3$ pc. Thus a given molecular cloud could contain many subunits which can collapse independently. Hence, if we assume that the type of collapse discussed is necessary for at least some chance of star formation we would conclude that it is likely that stars do not form individually but in groups within a parent cloud of high mass. This expectation is generally borne out by observation.

However, once a given cloud can start to contract, the story is not ended. Equation (8.27) shows that provided $c_{\rm c}$ remains constant (or decreases with increasing density) the critical mass for collapse decreases as the collapse proceeds (because the average density increases). We therefore expect that the initially collapsing cloud (which may be a subsection of a more massive cloud) becomes liable to break up into fragments of smaller mass which could themselves collapse independently. These fragments could become liable to fragmentation later on by the process by which they were formed.

This process of break-up into a hierarchy of fragments cannot continue indefinitely. As a cloud collapses, part of its gravitational potential energy can be converted into heat by compression or by the generation of supersonic motions which transform kinetic energy into heat via shock waves. If this energy input increases the sound speed then we can see from equation (8.27) that $M_{\rm crit}$ would be increased and the successive process of fragmentation ultimately will be halted. Generally, this occurs when the heat generated can no longer be radiated away efficiently. This will occur when the cooling radiation is unable to escape because of increasing opacity. Detailed investigation of this process suggests that the lowest mass it is possible to form by successive fragmentation is about $0.01 M_\odot$.

8.1.3 Induced star formation

We next consider clouds which are initially in equilibrium, now including an external surface pressure, and which therefore satisfy equation (8.8). This equation can be written in the form

$$P_{\rm s} = \frac{A}{R_{\rm c}^3} - \frac{B}{R_{\rm c}^4} \qquad (8.28)$$

where we have used equations (8.11) and (8.12). In equation (8.28), A ($\equiv 3kT_{\rm c}M_{\rm c}/4\pi\mu m_{\rm H}$) and B ($\equiv 3GM_{\rm c}^2/20\pi$) are constants for a cloud of a given mass $M_{\rm c}$ and temperature $T_{\rm c}$. A sketch of the variation of $P_{\rm s}$ with $R_{\rm c}$ is shown in figure 8.2.

If we fix the external pressure at some value P_0, the cloud can exist in either of two equilibrium states, E and D (figure 8.2). Suppose that the surface pressure is now raised to $P_0 + \Delta P_0$; we will briefly discuss later how this can happen. Figure 8.2 shows that, in principle, we move to new equilibrium states E′ and D′. But we must be cautious about this. First consider a cloud initially at D. Increasing the surface pressure physically must cause a cloud to contract, i.e.

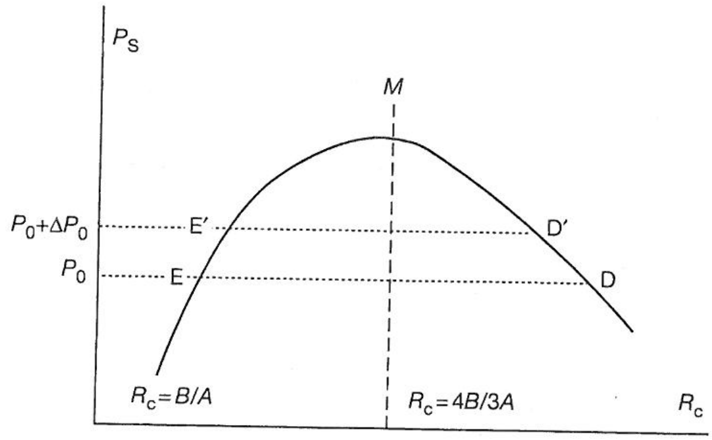

Figure 8.2. The pressure–radius relationship for an isothermal cloud with a surface pressure.

R_c must decrease. The internal pressure in the cloud increases to balance the external pressure. Hence the cloud moves from D $\rightarrow$ D′ with decreasing radius and increasing internal pressure. What happens if we start at E? If the external pressure is increased, figure 8.2 shows that we would have to increase the cloud radius in order to achieve a new equilibrium at E′. This is physically not possible, since the increased external pressure must cause the cloud to contract. Hence the cloud cannot stay in equilibrium and it starts to collapse. In other words, cloud equilibria to the left of the maximum M ($R_c < 4B/3A$) in figure 8.2 are unstable to collapse. Obviously, therefore, if we start with a stable configuration at D and increase the pressure sufficiently, we can cause clouds to move into an unstable regime. If this process leads to star formation, it is called 'induced' star formation, as opposed to 'spontaneous' star formation where a cloud collapses because it satisfies the contraction condition (8.10).

We have already met various dynamical processes which can increase the pressure of the interstellar medium, namely photoionization, supernovae explosions and stellar wind activity. All these may play a role in inducing star forming activity. The fact that the formation and dynamical effects of massive stars may induce further star formation gives rise to the concept of 'sequential star formation'. This process is sketched in figure 8.3. The existence of chains of O associations with the oldest at one end and the youngest at the other is strong evidence for these effects.

The picture of star formation as presented is simplified almost to the point of unreality. We have, for example, neglected the effects of the internal structure of the cloud. However, there are even more important omissions. The collapsing gas cloud will contain a magnetic field (which should be included in equation (8.8)). Collapse increases the magnetic field strength and hence the magnetic pressure and this will act, like thermal pressure, to oppose the collapse. The importance of magnetic fields depends critically on both the field geometry and how the field is coupled to the gas. It now seems clear that at some point the

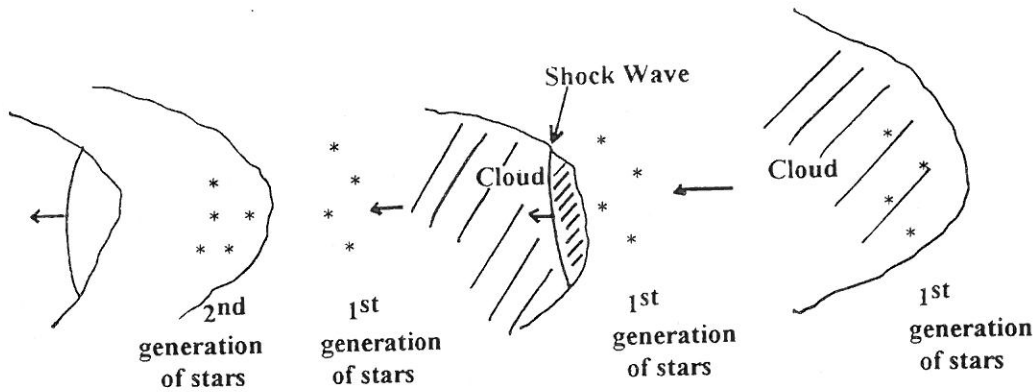

Figure 8.3. Sketch of sequential star formation in a cloud.

gas and field must decouple. Other complicating effects include those of cloud rotation since it is unlikely that any collapsing cloud will have zero angular momentum. In the absence of a braking torque, the rotational velocity of a cloud will increase because angular momentum is conserved. If the magnetic field is connected to surrounding gas, the necessary braking torque may be produced. All in all, star formation still remains one of the most fundamental of astrophysical problems.

8.2 Observational signatures of star forming activity

8.2.1 Infrared sources

The direct optical observation of young stars (or YSO's—Young Stellar Objects) in molecular clouds is extremely difficult, in practice almost impossible. The reason is simple: typical molecular clouds have optical depths in dust of 10–100. Hence the only way to investigate what is going on in star forming regions is in the infrared, millimetre wave and radio regions of the spectrum. Most of the progress in the study of star formation over the last two decades has come from the observational exploration of these spectral regions.

Although optical (or even harder) radiation cannot escape from the cloud, it can be converted into longer wavelength radiation and this can escape. This occurs because dust absorbs the direct short wavelength starlight and re-radiates it in the infrared because the grain temperature is low. We can crudely estimate the temperature, T_g, of a grain exposed to a flux of stellar radiation, F_R, by assuming that a grain absorbs and radiates as a black body (see also section 4.4.1). The grain temperature can then be estimated from

$$\sigma_R T_g^4 = F_R \tag{8.29}$$

where σ_R is the Stefan–Boltzmann radiation constant. If a grain is located at a