Astronomy 405: Solar System and the ISM (Spring 2013)
Homework 9, due on April 8

1. Photoionized HII regions usually have electron temperatures $\sim 10^4$ K. At such temperatures hydrogen's recombination coefficients are $\alpha_A = 4.18 \times 10^{-13}$ cm^3 s^{-1}, and $\alpha_B = 2.59 \times 10^{-13}$ cm^3 s^{-1}, and He$^+$'s recombination coefficient is $\alpha_B = 2.73 \times 10^{-13}$ cm^3. An O3V star has Q(H^0) = 7×10^{49} photons s^{-1} and Q(He^0) = 2.6×10^{49} photons s^{-1}. The interstellar medium around the star has a density of 100 H-atom cm^{-3}, and a H:He ratio of 10:1 (that is, 1 He atom for every 10 H atoms).

   (a) What is the mean free path of a photon with energy $h\nu$ =14 eV?

   (b) What is the mean free path of a photon with energy $h\nu$ =25 eV?

   (c) What is the size (in pc) of its HII region?

   (d) what is the mass (in $M_\odot$) of its HII region?

   (e) What is the size (in pc) of its HeII region?

   (f) Compare the sizes of its HII region and HeII region. If they are similar, calculate the HII region size again use $N_{\rm e} = N_{\rm H} + N_{\rm He}$.

2. Repeat Problem 1 for a B0III star with Q(H^0) = 1.3×10^{48} photons s^{-1} and Q(He^0) = 1.1×10^{46} photons s^{-1}.

   (a) What is the size (in pc) of its HII region?

   (b) what is the mass (in $M_\odot$) of its HII region?

   (c) What is the size (in pc) of its HeII region?

   (d) Compare the sizes of its HII region and HeII region. If they are similar, calculate the HII region size again use $N_{\rm e} = N_{\rm H} + N_{\rm He}$.

3. The energy level of a H-like atom is E_n =-13.6Z^2/n^2 eV, where Z is the electric charge of the nucleus. What transition does HeII λ4686Å line correspond to? How does the electron in He$^+$ get to this upper energy level?

4. Assume that the radius of a white dwarf $R_* = 1\ R_\oplus$, and use the Planck function, $B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT}-1}$, for the white dwarf's spectrum.

   (a) Calculate the He$^+$-ionizing flux, Q(He$^+$) = He$^+$-ionizing photons emitted per second, for white dwarfs of temperatures 25,000 K, 50,000 K, and 100,000 K. Note that Q(He$^+$) $= 4\pi R_*^2 \int \frac{\pi B_\nu}{h\nu}\, d\nu$, integrated from $h\nu = 54.5$ eV to ∞. (Use table of integrals, Mathematica or whatever numerical methods you like.)

   (b) If the white dwarf is in a nebula of density 1 H-atom cm^{-3}, what is the Strömgren radius of the He^{+2} zone? Do this for white dwarfs of the three temperatures above. (α_B(He$^+$) $\sim 2.5\times10^{-13}$ cm^3 s^{-1})