Astronomy 405 (Spring 2013)
Homework 6 (due on Mar 1)

Problem 1.
Comet X is a short period comet. Its perihelion is 0.8 AU from the Sun and its aphelion is 40 AU from the Sun, in the Kuiper Belt.
(a) What is the semi-major axis of this comet's orbit? (in units of AU)
(b) Show that Kepler's 3rd law for planets' orbits can be written as $a^3 = MP^2$, where a is the semi-major axis in AU, M is the mass of the central star in $M_{\rm Sun}$, and P is the orbital period in yr.
(c) Use the Earth's orbit to verify $a^3 = MP^2$.
(d) What is the orbital period of Comet X?

Problem 2.
You are riding a spaceship through the Asteroid Belt. Let's see how hard it is for you to duck the asteroids.
(a) The asteroid Belt extends from 2 AU to 3 AU from the Sun. If it has a thickness of 3 $R_{\rm Sun}$, what is the volume of the Asteroid Belt?
(b) If there are 400,000 asteroids, each with a radius of 50 km, what is the mean free path in the Asteroid Belt?
(c) What fraction of the total volume of the Asteroid Belt is actually occupied by asteroids?
(d) Have you played video games that involve blasting asteroids? Are their depictions of asteroid fields realistic? (If your answer to the first question is "no," you don't need to answer the second question, but you need a life…)

Problem 3.
An alien observes the Sun-Earth system from 1 pc away. See how difficult it is for the alien to detect the Earth.
(a) What is the luminosity ratio $L_{\rm Earth}/L_{\rm Sun}$?
(b) If the Sun-Earth orbit is in the alien's sky plane, what is the angular distance between the Sun and the Earth, as seen by the alien?
(c) The alien measures the Sun's position against the background quasars (stationary in the sky). What is the Sun's largest angular position shift?
(d) The alien has a telescope whose angular resolution is comparable to that of the HST, 0.05 arcsec. Is the alien able to detect the Sun's wobble and deduce the existence of a planet?
(e) If the Sun-Earth orbit is viewed edge-on, how much does the Sun's radial velocity vary due to its reflex motion? (Express the answer in m/s.)
(f) In a school zone, the speed limit is 20 miles/hour. Express this speed in units of m/s, and compare the speed with the Sun's velocity change in (e).
(g) What is the probability for the alien to see the Earth transiting the Sun?

Problem 4.

Let's estimate the energy generate rate of Asteroid Y by radioactive decay of ^{26}Al . Asteroid Y has a radius of 50 km. If it is heated only by sunlight, its equilibrium temperature should be 100 K, but it is observed to be 120 K. We assume that all the extra energy is generated internally by radioactive decay of ^{26}Al . Let's estimate the energy generate rate of Asteroid Y by radioactive decay of ^{26}Al . The half-life of ^{26}Al is 7.17×10^5 yr.

(a) What is the internal energy generation rate of Asteroid Y?

(b) The mass of ^{26}Al is 25.986892 u and the mass of ^{26}Mg is 25.982594 u. ^{26}Al decays into ^{26}Mg, and the difference in their mass is converted to energy. How much energy is released in each decay? (Express the answer in joules.)

(c) What is the ^{26}Al radioactive decay rate of Asteroid Y?

(d) How many ^{26}Al atoms are in Asteroid Y now?

(e) How many ^{26}Al atoms did Asteroid Y have 4.6 Gyr ago?