Astronomy 405 (Spring 2013)
Homework 5 (due on Feb 22)

Problem 1.

Dust grains orbiting around the sun absorb sunlight and re-emit photons. From a rest frame, a dust grain is moving and radiating preferentially in the forward direction. As a photon has an effective mass of E/c^2, the dust grain essentially loses angular momentum when emitting photons. If a dust grain loses angular momentum, its orbital radius would decrease and the dust grain will spiral in. This is called Poynting-Robertson effect.

(a) If a dust grant has zero albedo, and radiates all energy received from sunlight. For a cross-section of σ_g and a distance of r from the Sun, what is the rate of energy radiated by the dust grain?

(b) Show that the rate of a dust grain's angular momentum loss is given by

$$\frac{d\mathcal{L}}{dt} = -\frac{\sigma_g}{4\pi r^2}\frac{L_\odot}{mc^2}\mathcal{L},$$

(c) If the dust grain has a radius of R and density of ρ, show that the time for a dust grain to spiral into the Sun is

$$t_{\text{Sun}} = \frac{4\pi\rho c^2}{3L_\odot}Rr^2$$

(d) For a dust grain of density 2.5 g cm^{-3} and radius 0.1 mm, how long does it take for the dust grain to spiral from 1 AU to the Sun's surface?

(e) If a dust grain is orbiting around Saturn, but its dominant heating source is the Sun. Start from the equation in (b), substitute r by r_s, Saturn's distance to the Sun. Since the dust grain orbits around Saturn, you can treat r_s as a constant. Express the angular momentum as a function of r, the distance to Saturn, and substitute it into the above angular momentum equation. Show that the dust grain will spiral from its initial orbital radius R_0 to the surface of Saturn (radius R_S) in time

$$t_{\text{Saturn}} = \frac{8\pi\rho c^2}{3L_\odot}Rr_S^2 \ln\left(\frac{R_0}{R_S}\right)$$

(f) For a dust grain of radius 10 μm and density of 1 g cm^{-3}, how long does it take to spiral from 6 R_S to the surface of Saturn?

(g) Compare the answer of (e) to the age of the solar system, ~4.6 Gyr. What does this imply on the origin of the dust? (i.e., was it remnant from the formation or replenished?)

Problem 2.

The central star of the Helix Nebula is a white dwarf with an effective temperature of 110,000 K, a radius of 7 times Earth radius, and a mass of 0.6 solar masses. This white dwarf is surrounded by a dust disk. Assuming that the dust grain has a density of 2.5 g cm^{-3}, what is the smallest dust grain that can remain in the dust disk without being driven out by the radiation pressure?

Problem 3.

A star is surrounded by a dust disk at radii 30-80 AU. Assume that all dust grains are spheres of radius 0.1mm and density 2.5 g cm^{-3}. The dust disk has an optical depth of 1 radially along the disk; that is, (disk width) * (cross section of a dust grain) * (number density of dust grains) = 1.

(a) What is the number density of dust grain in the dust disk?

(b) If the total mass of the dust disk is 0.1 Earth mass, what is the thickness of the dust disk?

Problem 4.

Use the gas and dust discharge rates of Comet Halley on page 823 of Carroll and Ostlie, and assume that these discharges last for a year at perihelion.

(a) How much mass does Halley lose during each visit to the Sun?

(b) Halley's nucleus has a mass of $\sim 5 \times 10^{13}$ kg. For an orbital period of 75 years, in how many more years will Halley become extinct?