Astronomy 405 (Spring 2013)
Homework 3 (due on Feb 8)

Problem 1.

Use a uniform solid body approximation for the Earth. Its moment of inertia is $I = (2/5)MR^2$, and its rotational kinetic energy is $K = (1/2)I\,\omega^2$.

(a) Due to tidal friction, the Earth's rotation is slowing down and its rotational period is lengthening at a rate of 0.0016 s/century. What is the rate of rotational energy release of the Earth?

(b) If the rotational energy release is radiated away as heat uniformly over the surface of the Earth, what is the average flux (in units of W m^{-2})?

(c) How much heating from the Sun does the Earth get (in units of W)?

(d) Compare the answers from (a) and (c). Which source is more important?

Problem 2.

Optical depth can be thought of as the number of mean free paths from the original position to the surface. For a planet atmosphere with optical depth τ, you can divide the atmosphere into τ layers with a unity optical depth in each layer. Each layer only receives the emission from the adjacent layers.

(a) What is the temperature at the top of this planet's atmosphere at subsolar location? Assume no circulation of air to distribute heat over the entire surface. Express T_{top} in terms of stellar luminosity L_*, distance to the star D, and albedo a.

(b) Show that the temperature on the surface of the planet (under an atmosphere of optical depth τ) is $T_{surf} = (1 + \tau)^{1/4}\, T_{top}$.

Problem 3.

Planet X orbits around the Sun with an orbital period identical to its rotational period. Assuming no atmosphere on Planet X. What is the temperature at position A (see the diagram above)? Express the answer in terms of the Sun's $R_\odot$ and $T_\odot$ and Planet X's orbital radius D and albedo a.

${}^{147}_{62}Sm/{}^{144}_{60}Nd$	${}^{143}_{60}Nd/{}^{144}_{60}Nd$
0.1847	0.511721 ± 18
0.1963	0.511998 ± 16
0.1980	0.512035 ± 21
0.2061	0.512238 ± 17
0.2715	0.513788 ± 15
0.2879	0.514154 ± 17

Problem 4.

${}^{147}{}_{62}Sm$ decays into ${}^{143}{}_{60}Nd$ with a half-life of 1.06×10^{11} yr. The abundances of these atoms can be used to date the age of a rock. The Table above shows abundance ratios ${}^{147}{}_{62}Sm/{}^{144}{}_{60}Nd$ and ${}^{143}{}_{60}Nd/{}^{144}{}_{60}Nd$ measured from different parts of a Moon rock.

(a) What kind of decay turns ${}^{147}{}_{62}Sm$ into ${}^{143}{}_{60}Nd$?

(b) Show that
$N({}^{143}{}_{60}Nd) / N({}^{144}{}_{60}Nd) = (e^{\lambda t} - 1)\, N({}^{147}{}_{62}Sm)/ N({}^{144}{}_{60}Nd) + N_i({}^{143}{}_{60}Nd) / N({}^{144}{}_{60}Nd)$
where N(X) is the current number of the atoms of species X, N_i (X) is the initial number of atoms of species X, and λ is $\ln 2 / \tau_{1/2}$ and $\tau_{1/2}$ is the half-life of ${}^{147}{}_{62}Sm$.

(c) Graph ${}^{147}{}_{62}Sm/{}^{144}{}_{60}Nd$ against ${}^{143}{}_{60}Nd/{}^{144}{}_{60}Nd$ using the data from the Table above.

(d) Use the plot in (c) to determine the age of the Moon rock.

(e) Why do we bother measuring ${}^{144}{}_{60}Nd$ and use the abundance ratios in this experiment?