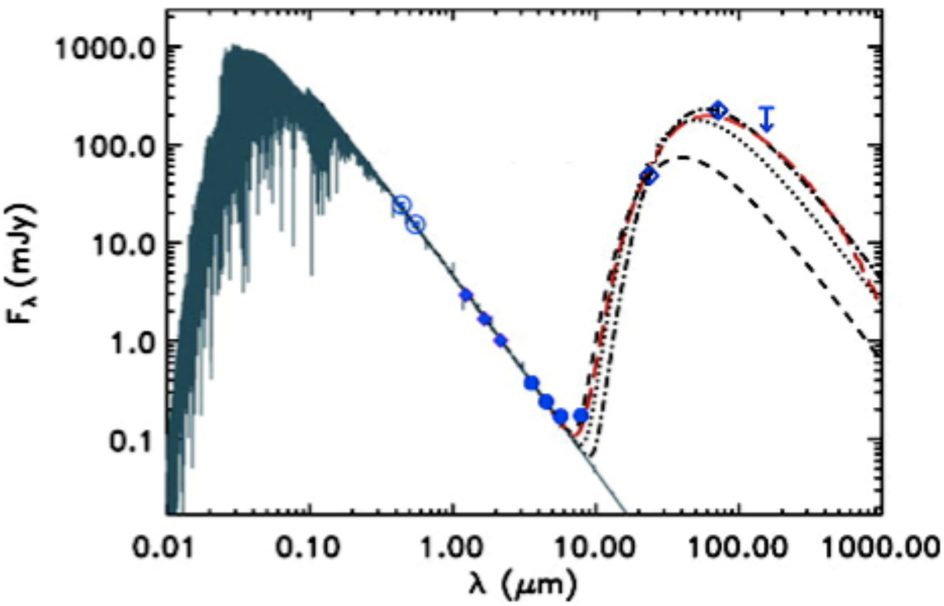
**Astronomy 405 (Spring 2013)**
**Homework 2 (due on Feb 1)**

**Problem 1.**
The spectral energy distribution of the central star of the Helix Nebula is show in the figure below. The UV and optical emission from the star's photosphere contributes to the component to the left. We want to figure out the origin of the IR emission.




(a) The star has a luminosity of 76 $L_\odot$ and its photospheric emission peaks at ~0.025 μm. Calculate the effective temperature and radius of this star.
(b) The luminosity of the IR emitter is $8 \times 10^{31}$ erg/s, and the emission peaks at ~40 μm. Calculate the effective temperature of this emitter and its emitting surface area.
(c) Compare the temperature and surface area of this IR emitter to those of stars (Appendix G) and planets (Appendix C) and show that only a dust cloud can provide such temperature and emitting area.
(d) Assuming that the dust has an albedo of 0.2, at what distance from the star are the dust grains located?
(e) In the solar system, what objects are located at such distance from the Sun?
(f) As the progenitor of the central star of the Helix Nebula evolved and lost mass (it went from 3 $M_\odot$ to 0.6 $M_\odot$), how would its planetary system adjust its kinematics? It has been suggested that this adjustment caused more collisions among objects similar to those in (e) and produced the dust.

**Problem 2.**

(a) Planet X has a rotation period that equals its orbital period around the Sun. There is no circulation of atmosphere to equalize the temperature on the surface of Planet X. What is the temperature at the subsolar point? [Express it in terms of $R_\odot$, $T_\odot$, distance to the Sun ($D$), and albedo $a$.]

(b) Planet Y has a thick atmosphere that is totally transparent to the sunlight but totally opaque to the IR emission from the surface of the planet. Show that the surface temperature of Planet Y is

$T_{surf} / T_\odot = 2^{-1/4} (1 - a)^{1/4} (R_\odot / D)^{1/2}$

**Problem 3.**

Start from the following equation

$$\dot{N} = 4\pi R^2 \left(\frac{1}{16}\right) n \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{v_{esc}}^{\infty} 4\pi v^3 e^{-mv^2/2kT} dv$$

and use integration by parts to show that

$$\dot{N}(z) = 4\pi R^2 \nu n(z)$$

$$\nu = (1/8)\left(\frac{m}{2\pi kT}\right)^{1/2} (v_{esc}^2 + 2kT/m)\, e^{-mv_{esc}^2/2kT}$$

.

**Problem 4.**

(a) The mean free path of a particle is $(n\,\sigma)^{-1}$, where n is the number density of particles and σ is the cross section (= $3.52 \times 10^{-20}$ m$^2$ for H atom). If the mean free path in the exosphere has to be 400 km in order for a hydrogen atom to escape, what is the number density of H atom in the exosphere?

(b) What is the escape velocity in the Earth's exosphere? [You can use the Earth's radius, since the exosphere is so much thinner than the Earth's radius.]

(c) Calculate the escape parameter ν you derived in problem 3, assuming the temperature at the exosphere is 1000 K.

(d) If the hydrogen density in the exosphere is $10^{11}$ m$^{-3}$, what is rate of hydrogen atom loss from the exosphere?

(e) Suppose there were as many hydrogen atoms in a young Earth's atmosphere as the number of nitrogen molecules in the atmosphere now, $N = 9 \times 10^{43}$, how long does it take to lose all the hydrogen atoms?

(f) Compare the answer in (e) to the age of the Earth, $4.6 \times 10^9$ yr. There is hardly any hydrogen in the Earth's atmosphere now. Does this mean hydrogen atoms have escaped or the young Earth did not have this much hydrogen in its atmosphere?