Astronomy 405 (Spring 2013)
Homework 1 (due on Jan 25)

1. The Earth's orbital radius is 1 AU. Suppose Planet X also orbits around the Sun in the ecliptic plane. From the Earth it is observed that Planet X is at most 10 degrees from the Sun in the sky.
   (a) What is the orbital radius of Planet X in units of AU?
   (b) If Planet X is as large as the Earth, how does the tidal force on Planet X (due to the Sun) compare to that on the Earth?
   (c) Repeat (b), if Planet X's radius is 1/10 of the Earth's radius.

2. Moon Rio, whose mass is *m,* orbits around Planet X, whose mass is *M.* Assume that *M* is much larger than *m.* Moon Rio's orbital radius is *r*.
   (a) Show that the orbital period of Moon Rio is $P = (4\,\pi^2\, r^3 / G\, M)^{1/2}$. [Hint: start by balancing the centrifugal and gravitational forces.]
   (b) If Planet X has a rotational period of p and Moon Rio has a synchronous orbit, what is its orbital radius?
   (c) Planet X has $M = 6 \times 10^{24}$ kg and $p = 10$ days. What is the radius of a synchronous orbit?
   (d) If Moon Rio has a prograde orbit, under what condition will it spiral in and under what condition will it spiral out?
   (e) Repeat (d), if Moon Rio has a retrograde orbit.

3. A neutron star with a mass of 5×10^{30} kg and a radius of 1×10^4 m.
   (a) If the Earth wandered close to a neutron star, at what distance will the Earth be destroyed by the tidal force?
   (b) For an asteroid with an average density of 2 g/cm^3 and a radius of 100 km, at what distance to the neutron star will the asteroid breakup?
   (c) For a gas cloud with an average density of 1000 H-atom/cm^3 and a radius of 1 pc, at what distance to the neutron star will the cloud be shredded?

4. Assume that the Earth has a uniform density. Its rotation is slowing down and its rotation period is increasing at a rate of 0.0016 s per century.
   (a) What is the rate of change in the Earth's rotational angular momentum?
   (b) The Moon is drifting away from the Earth by 3.5 cm per year. What is the rate of change of the Moon's angular momentum?
   (c) Compare the rates of change in (a) and (b), what do you find about the total angular momentum?