Astronomy 405: Solar System and the ISM (Spring 2013)
Homework 10, due on April 15

1. (a) Show that the collisional de-excitation rate is $N_e\ N_2\ q_{21}\ =\ N_e\ N_2\ \frac{8.629\times10^{-6}}{T^{1/2}}\ \frac{\Omega(1,2)}{\omega_2}$

   (b) Show that the collisional excitation rate is $N_e\ N_1\ q_{12}\ =\ N_e\ N_1\ \frac{8.629\times10^{-6}}{T^{1/2}}\ \frac{\Omega(1,2)}{\omega_1}\ e^{-\chi/kT}$. [Hint: Follow the derivation of the collisional de-excitation rate. Integrate only from the threshold velocity to infinity. Use the definitions given in the lecture notes.]

2. Show that the critical density of OIII $^1D_2$ is $N_{\rm e} = 7 \times 10^5$ cm^{-3}.

3. The Hα emission from an optically-thick nebula can be used to determine properties of its ionizing source.

   (a) The Tarantula Nebula (30 Doradus) has an Hα luminosity of 1.5×10^{40} ergs s^{-1}. What is the required ionizing flux of the Tarantula Nebula?

   (b) Can the Tarantula Nebula be ionized by a single star? If not, how many O5 stars are needed to provide this ionizing flux? (O5 star has Q(H^0) = $10^{49.67}$ photons/s)

   (c) Show that roughly every 3.5 recombinations of hydrogen lead to the emission of 1 Hα photon.

4. The Table below gives spectra of three ionized nebulae.

   (a) Use the [O III] lines and the [N II] lines, if available, to determine electron temperatures for Neb 1, Neb 2, and Neb 3.

   (b) Use [O II] and [S II] lines to determine electron densities of Neb 2.

   (c) Neb 3 has tricky line ratios. Let's assume that Neb 3 has an electron temperature of 20,000 K. What is the electron density of Neb 3?

   (d) Why does Neb 3 have such weak [O III]λ5007 line?

λ (Å)	Line ID	Nebula 1	Nebula 2	Nebula 3
3726	[O II]	–	19	–
3729	[O II]	–	22	–
4363	[O III]	8	1.2	13
4686	He II	–	–	7
4861	Hβ	100	100	100
4959	[O III]	112	74	5
5007	[O III]	317	223	17
5755	[N II]	–	2.2	9
6548	[N II]	16	72	5
6563	Hα	281	285	285
6583	[N II]	48	229	14
6717	[S II]	<11	7.5	–
6731	[S II]	<8	6.8	–