Astronomy 404
September 16, 2013

Chapter 8. The Classification of Stellar Spectra

What determines the spectral line strengths?

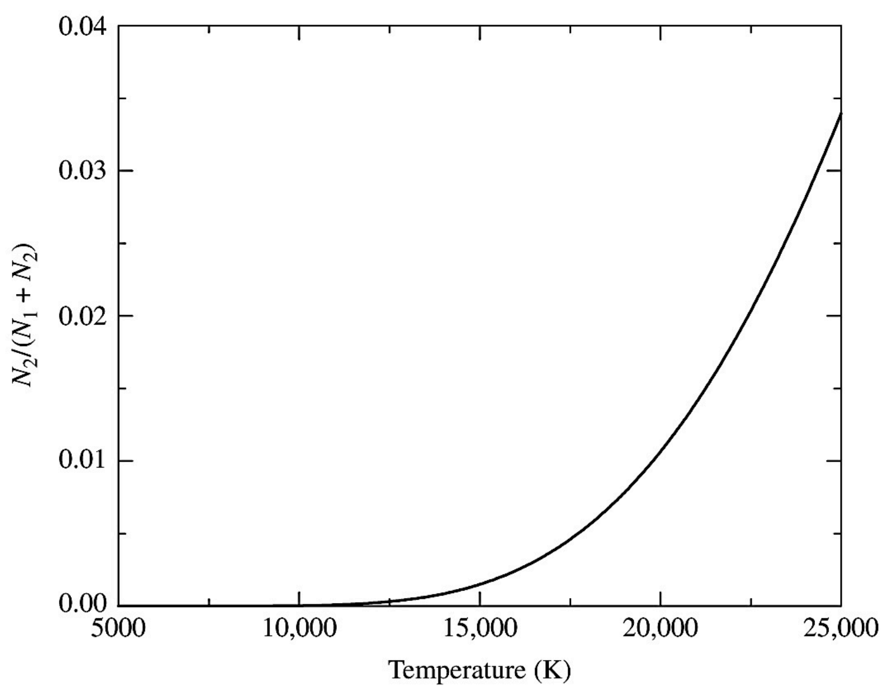
- Maxwell-Boltzmann velocity distribution

$$n_v \, dv = n \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} \, 4\pi v^2 \, dv,$$

At *thermodynamic equilibrium* :

- Boltzmann Equation - population at different energy levels

$$\frac{N_b}{N_a} = \frac{g_b \, e^{-E_b/kT}}{g_a \, e^{-E_a/kT}} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}.$$



- Saha Equation - population at different ionization stages

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.$$

where Z is the partition function, a weighted sum of the numbers of states (statistical weights g) of all energy levels:

$$Z = \sum_{j=1}^{\infty} g_j\, e^{-(E_j - E_1)/kT}.$$

A free electron have two possible spins, $m_s = \pm 1/2$, thus "2" in Saha Eq.

For hydrogen, HI → HII, χ_I = 13.6 eV.

$$Z_{\mathrm{I}} = 2 + 2 \times 2^2 \times e^{(\frac{13.6}{2^2} - 13.6)/kT} + 2 \times 3^2 \times e^{(\frac{13.6}{3^2} - 13.6)/kT} + \ldots$$

$$2 \times 2^2 \times e^{(\frac{13.6}{2^2} - 13.6)/kT} = 4\times10^{-10} \text{ at } T = 5000\text{ K}$$
$$= 0.07 \text{ at } T = 25{,}000\text{ K}$$

Therefore, $Z_{\mathrm{I}} \approx g_1 \approx 2$.

[Note: When $n \to \infty$, $2\times n^2 \times e^{(\frac{13.6}{n^2} - 13.6)/kT} \to 2\times n^2 \times e^{-13.6/kT}$.
The sum diverges, but when $n \to \infty$, the atomic orbitals overlap with neighboring atoms, and the electrons are no longer associated with a single nucleus.]

$Z_{\mathrm{II}} = 1$.

Saha equation:

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.$$

Sometimes the pressure of the free electrons, P_e, is used in place of the electro density n_e. Applying the ideal gas law $P_e = n_e k\,T$, the Saha equation can be written as:

$$\frac{N_{i+1}}{N_i} = \frac{2kT\,Z_{i+1}}{P_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}.$$

The electron pressure P_e ranges from 0.1 N m^{-2} for atmospheres of cooler stars to 100 N m^{-2} for hotter stars.

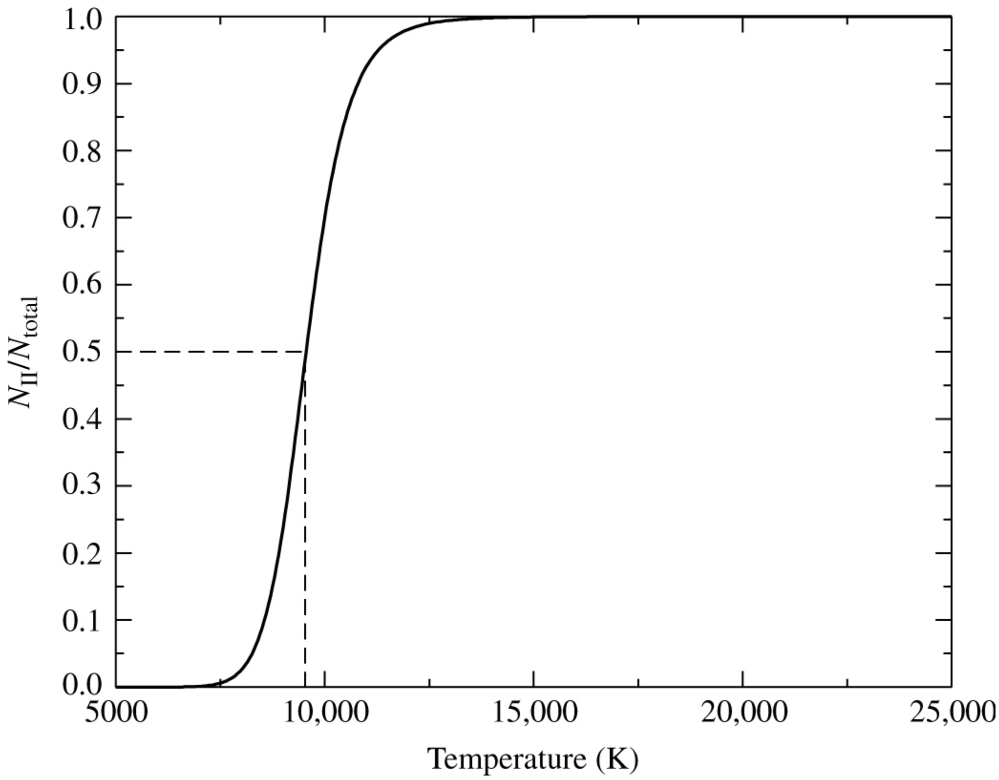
For a pure hydrogen atmosphere with P_e = 20 N m^{-2} ,

$N_{II} / N_{total} = N_{II} / (N_I + N_{II}) = \frac{N_{II}/N_I}{1+ N_{II}/N_I}$

N_{II} / N_I can be calculated from the Saha equation.

N_{II} / N_{total} = 0.05 at T = 8,300 K
= 0.5 at T = 9,600 K
= 0.95 at T = 11,300 K

8,300 – 11,300 K or **~ 10,000 K**
“Partial Ionization Zone” for hydrogen.



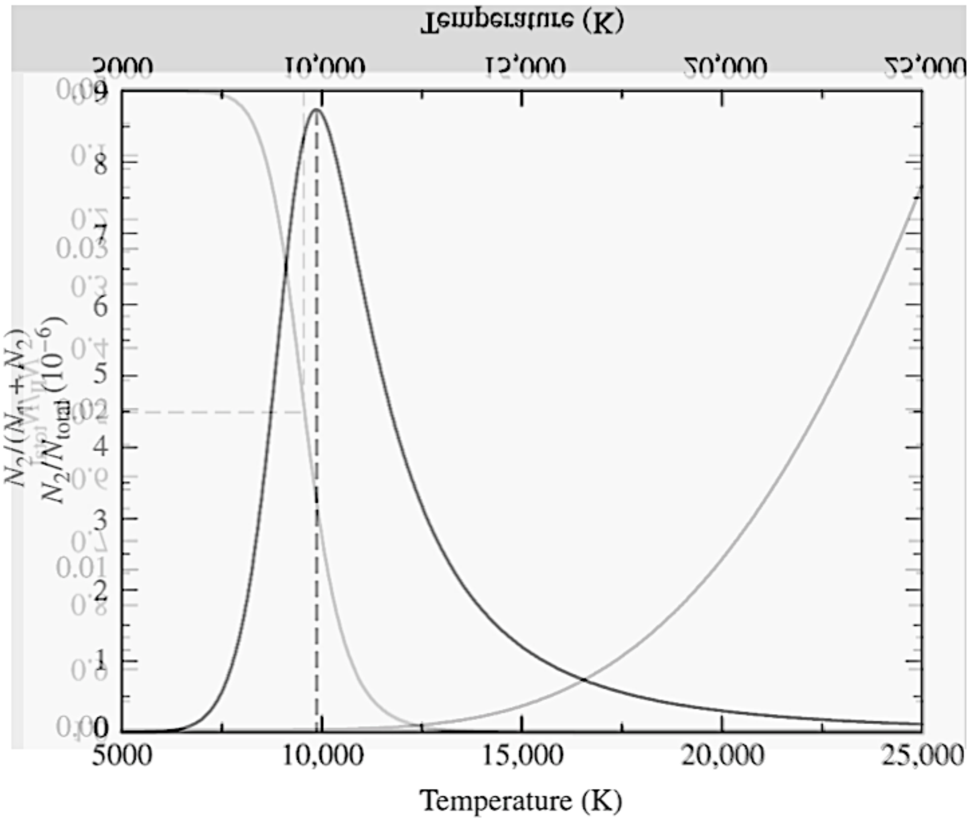
Balmer line strengths depends on N_2 (number of HI at n = 2).

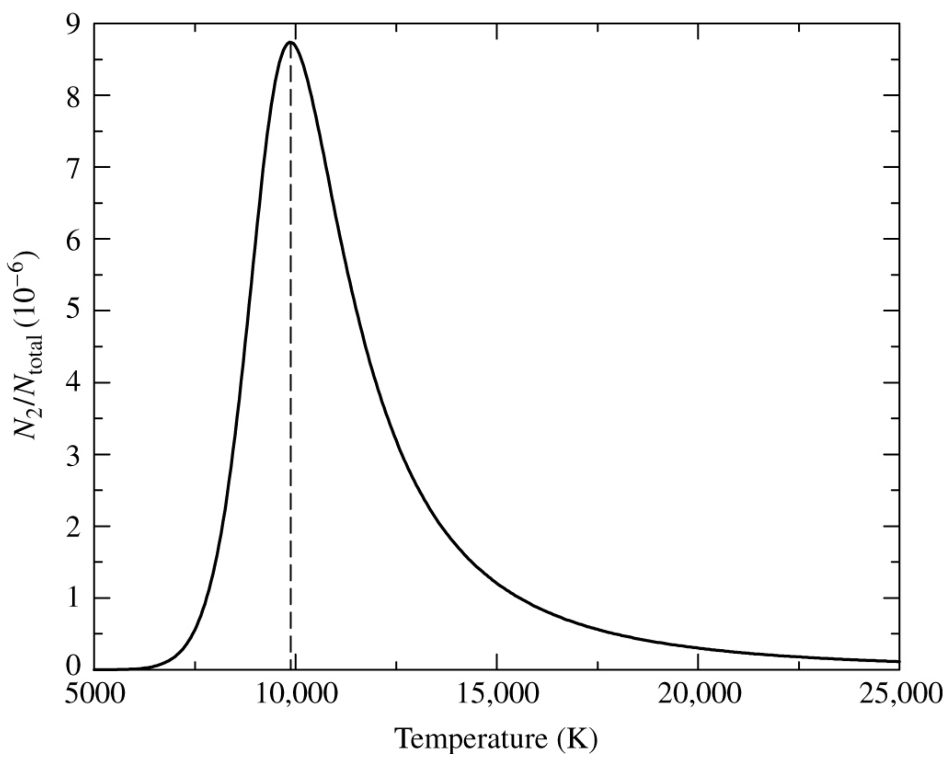
$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_I}{N_{total}}\right) = \left(\frac{N_2/N_1}{1 + N_2/N_1}\right)\left(\frac{1}{1 + N_{II}/N_I}\right)$$

N_2 / N_1 from Boltzmann Equation
N_{II} / N_I from Saha Equation

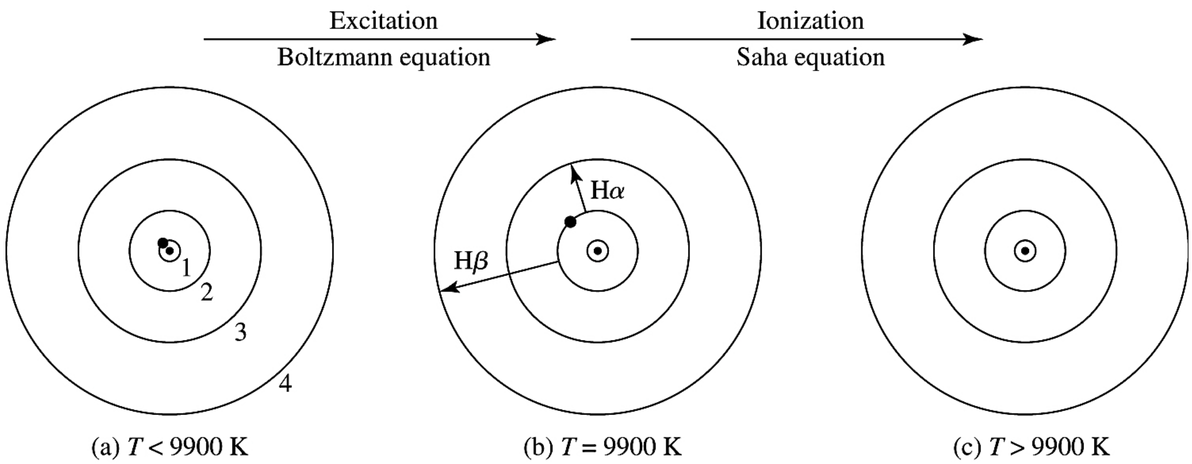
Caution: Saha equation is valid only if the gas is in thermal equilibrium and Maxwell-Boltzmann velocity distribution is obeyed. Density cannot be too high (< 1 kg/m^3), or the neighboring ions will interfere with the electron orbitals.

Earth air at sea level at ~300 K has a density of ~1.2 kg/m^3.





***N_2/N_{total}* Peaks at 9900 K, ~ A0 star.**



Why are the CaII H+K lines stronger than Balmer lines in the solar spectrum?

$T = T_e = 5777$ K

$P_e = 1.5$ N m^{-2}

$N_{Ca} : N_H = 1 : 500{,}000$

Saha equation:

$$\left[\frac{N_{II}}{N_I}\right]_H = \frac{2kT\,Z_{i+1}}{P_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT} = 7.70 \times 10^{-5} \simeq \frac{1}{13{,}000}$$

Boltzmann equation:

$$\left[\frac{N_2}{N_1}\right]_{H\,I} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = 5.06 \times 10^{-9} \simeq \frac{1}{198{,}000{,}000}$$

Therefore,

$$\frac{N_2}{N_{total}} = \left(\frac{N_2}{N_1 + N_2}\right)\left(\frac{N_I}{N_{total}}\right) = 5.06 \times 10^{-9}$$

for hydrogen.

Now Calcium.

Ca I has an ionization energy χ_I = 6.11 eV, less than ½ that of H I. This difference in χ_I makes a big difference in Saha equation, because $kT \sim$ 0.5 eV, and $e^{-13.6/0.5} << e^{-6.11/0.5}$.

Ca Z_I = 1.32, Z_{II} = 2.30

Saha equation:

$$\left[\frac{N_{II}}{N_I}\right]_{Ca} = \frac{2kT Z_{II}}{P_e Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT} = 918.$$

Practically all Ca atoms are in the form of Ca II !!!

Ca II H & K lines are 396.8 nm & 393.3 nm
$E_2 - E_1 = hc/\lambda = 3.12$ eV
$g_1 = 2$, $g_2 = 4$
Boltzmann equation:

$$\left[\frac{N_2}{N_1}\right]_{Ca\ II} = \frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = 3.79 \times 10^{-3} = \frac{1}{264}$$

The great majority of Ca II atoms are in the ground state.

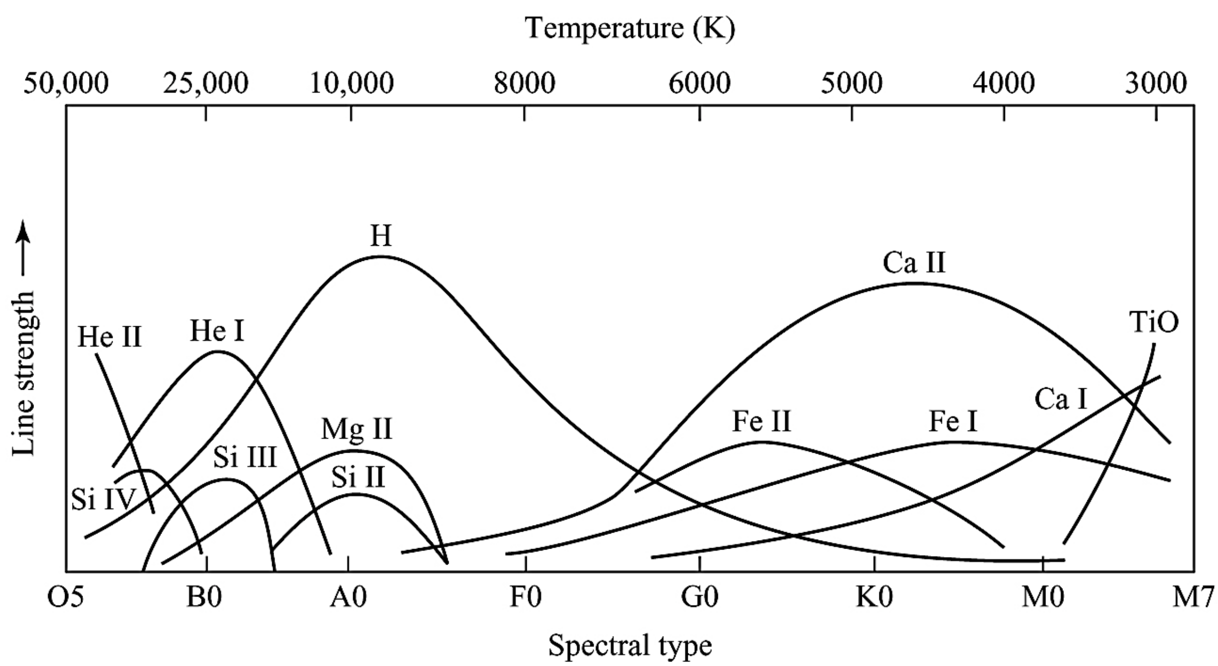
$$\left[\frac{N_1}{N_{total}}\right]_{Ca\ II} \simeq \left[\frac{N_1}{N_1+N_2}\right]_{Ca\ II} \left[\frac{N_{II}}{N_{total}}\right]_{Ca}$$
$$= \left(\frac{1}{1+[N_2/N_1]_{Ca\ II}}\right)\left(\frac{[N_{II}/N_I]_{Ca}}{1+[N_{II}/N_I]_{Ca}}\right)$$
$$= \left(\frac{1}{1+3.79\times10^{-3}}\right)\left(\frac{918}{1+918}\right)$$
$$= 0.995.$$

99.5% of Ca atoms are in Ca II ground state, ready to absorb.

# of H atoms ready for Balmer lines / # of Ca atoms ready for H&K

= (500,000) × (5.06×10^{-9}) ≈ 000253 = 1/395

Therefore, Ca II H&K lines in the solar spectrum are much stronger than the hydrogen Balmer lines.



The first person to determine the composition correctly was Cecilia Payne (1900-1979). In her 1925 PhD thesis, she determined relative abundances of 18 elements in stellar atmospheres.

Stellar Atmospheres

Stellar Atmospheres

- How does energy propagate through and emerge from the surface of a star?
- Star composed of layer upon layer of small packets of gas:
  - absorbs light from lower layers
  - emits light into upper layers
  - moves up or down carrying energy
  - collide with other packets transferring energy
  - ...

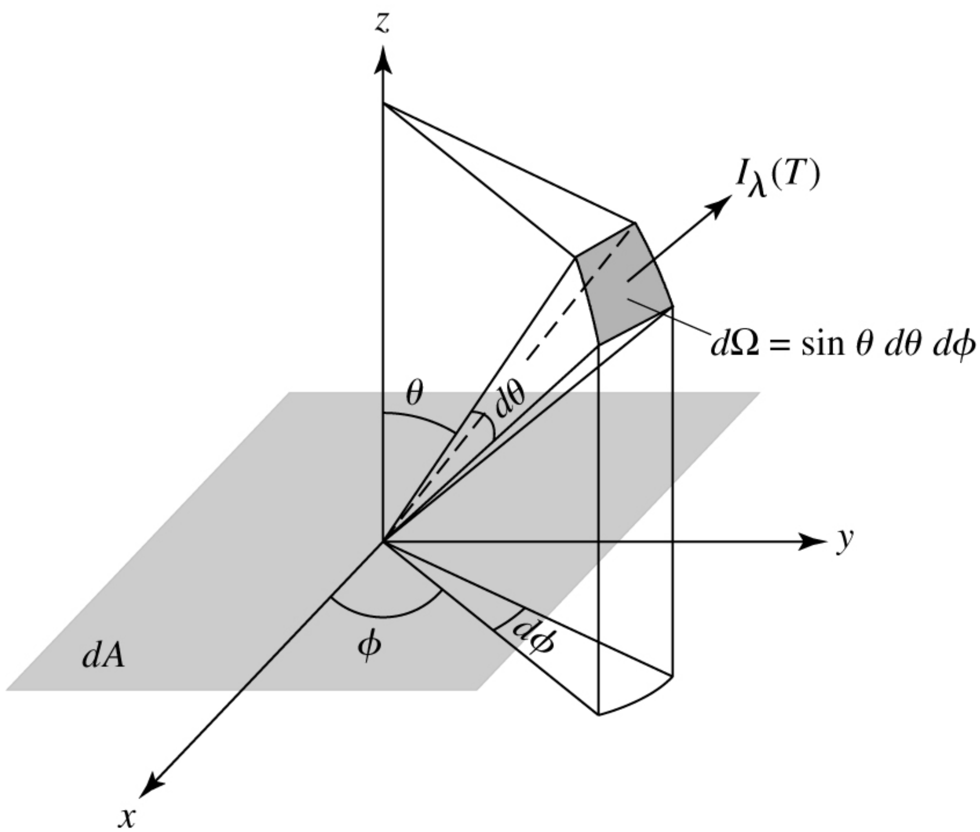
Solid Angle

- Solid angle: 2-D analog of an angle — apex of a cone.
- 1-D angle → arc length, s; solid angle → surface area, A:

$$r\,d\theta = ds \qquad r^2 d\Omega = dA$$

- Unit: steradian, sr — 4π sr in a spherical surface.
- Small element of solid angle, $d\Omega$, in spherical coordinates:
  - Side 1 has length $d\theta$
  - Side 2 has length $\sin\theta d\phi$
  - Area $d\Omega = \sin\theta d\theta d\phi$

$$\int d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta d\theta d\phi = \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi = 2\cdot 2\pi$$



$E_\lambda\, d\lambda$ is the amount of energy that the light rays carry into the cone in a time interval *dt.*

$$E_\lambda \equiv \frac{\partial E}{\partial \lambda},$$

The specific intensity of the rays is defined to be:

$$I_\lambda \equiv \frac{\partial I}{\partial \lambda} \equiv \frac{E_\lambda\, d\lambda}{d\lambda\, dt\, dA\, \cos\theta\, d\Omega}.$$

$$E_\lambda\, d\lambda = I_\lambda\, d\lambda\, dt\, dA\, \cos\theta\, d\Omega = I_\lambda\, d\lambda\, dt\, dA\, \cos\theta\, \sin\theta\, d\theta\, d\phi$$

The unit of specific intensity is W m^{-3} sr^{-1} . The Planck function B_λ is an example of specific intensity.