Astronomy 404
September 13, 2013

Chapter 8. The Classification of Stellar Spectra

How are spectral lines formed?

Atoms making transitions between energy levels.
A spectral-line photon can be absorbed only if an atom is at the right ionization stage and energy level.

How are the energy levels populated?

- Statistical mechanics
  Individual particle's behavior may be chaotic, but the temperature, pressure, and density of the gas are well-defined.
- Maxwell-Boltzmann velocity distribution - collisions
- Boltzmann Equation - population at different energy levels
- Saha Equation - population at different ionization stages

Maxwell-Boltzmann velocity distribution function

The number of gas particles per unit volume having speeds between *v* and *v*+*dv* is given by

$$n_v\,dv = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\,4\pi v^2\,dv,$$

where *n* is the total number density, *m* is the particle mass, *k* is Boltzmann's constant, *T* is the temperature, $n_v \equiv \partial n/\partial v$.

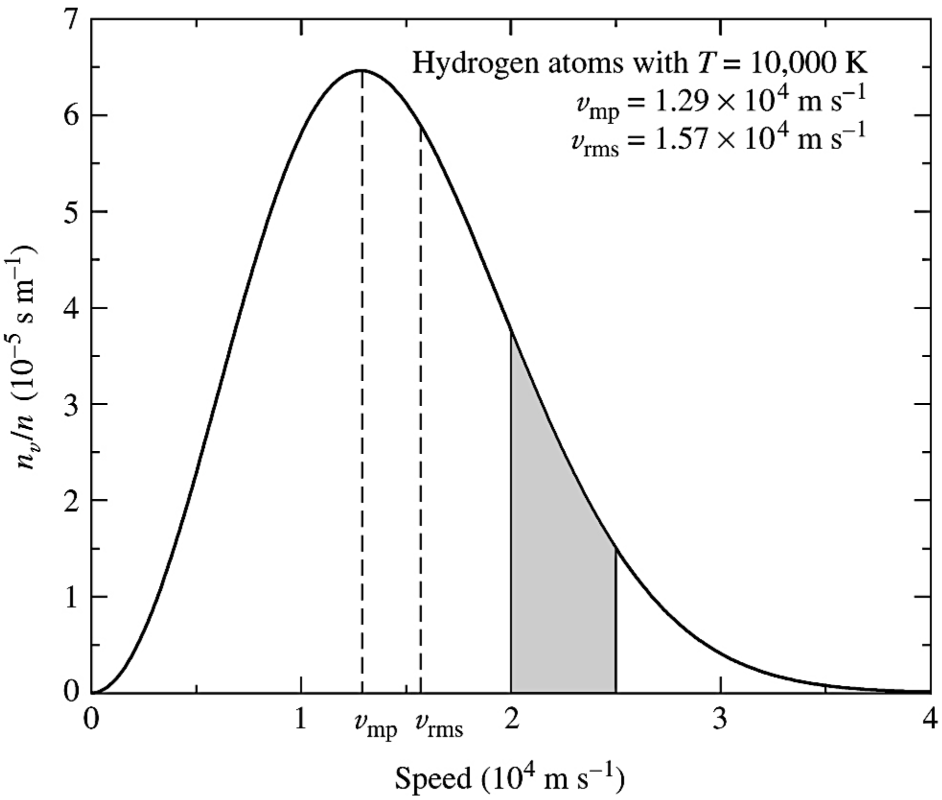


Figure 8.6 Maxwell-Boltzmann distribution function for H atoms at T = 10,000 K.

Most probable speed:

$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

root-mean-square speed:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

To determine the fraction of gas particles in the velocity range v_1 to v_2 , just integrate the Maxwell-Boltzmann distribution between the two velocity limits.

Boltzmann Equation

Atoms of a gas gain or lose energy as they collide. Given the Maxwell-Boltzmann velocity distribution, the collisions will produce a definite distribution of electrons among the atomic orbitals (i.e., energy levels).

Each state is defined by a set of quantum numbers n, ℓ, m_l, m_s.

States s_a and s_b are at energies E_a and E_b.
The ratio of the probability $P(s_b)$ that the system is in state s_b to the probability $P(s_a)$ that the system is in state s_a is given by

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}} = e^{-(E_b-E_a)/kT}$$

(The term $e^{-E/kT}$ is called the Boltzmann factor.)

$T \rightarrow \infty \qquad P(s_b)/P(s_a) \rightarrow 1$

$T \rightarrow 0 \qquad P(s_b)/P(s_a) \rightarrow 0$

However, several quantum states may have the same energy level. The energy level is degenerate. $s_a \neq s_b$, but $E_a = E_b$.

g_a: the number of states with E_a, the statistical weight of E_a.
g_b: the number of states with E_b, the statistical weight of E_b.

$$\frac{P(E_b)}{P(E_a)} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT}$$

Stellar atmospheres have lots of atoms, so the ratio of probabilities is the same as the ratio of number of atoms.

Boltzmann Equation is thus:

$$\frac{N_b}{N_a} = \frac{g_b\, e^{-E_b/kT}}{g_a\, e^{-E_a/kT}} = \frac{g_b}{g_a}\, e^{-(E_b-E_a)/kT} .$$

The statistical weight of hydrogen atom at energy level n is $g_n = 2n^2$.

Derive this. If $g_n = 2n^2$ holds when $n = 1$ & 2 and if $g_n = 2n^2$ holds for $n+1$ when it holds for n, then $g_n = 2n^2$ must hold for all n . Remember that for each n,

$\ell = 0, 1, \ldots, n-1$

$m_l = -\ell, -\ell+1, \ldots, \ell-1, \ell$

$m_s = \pm 1/2$

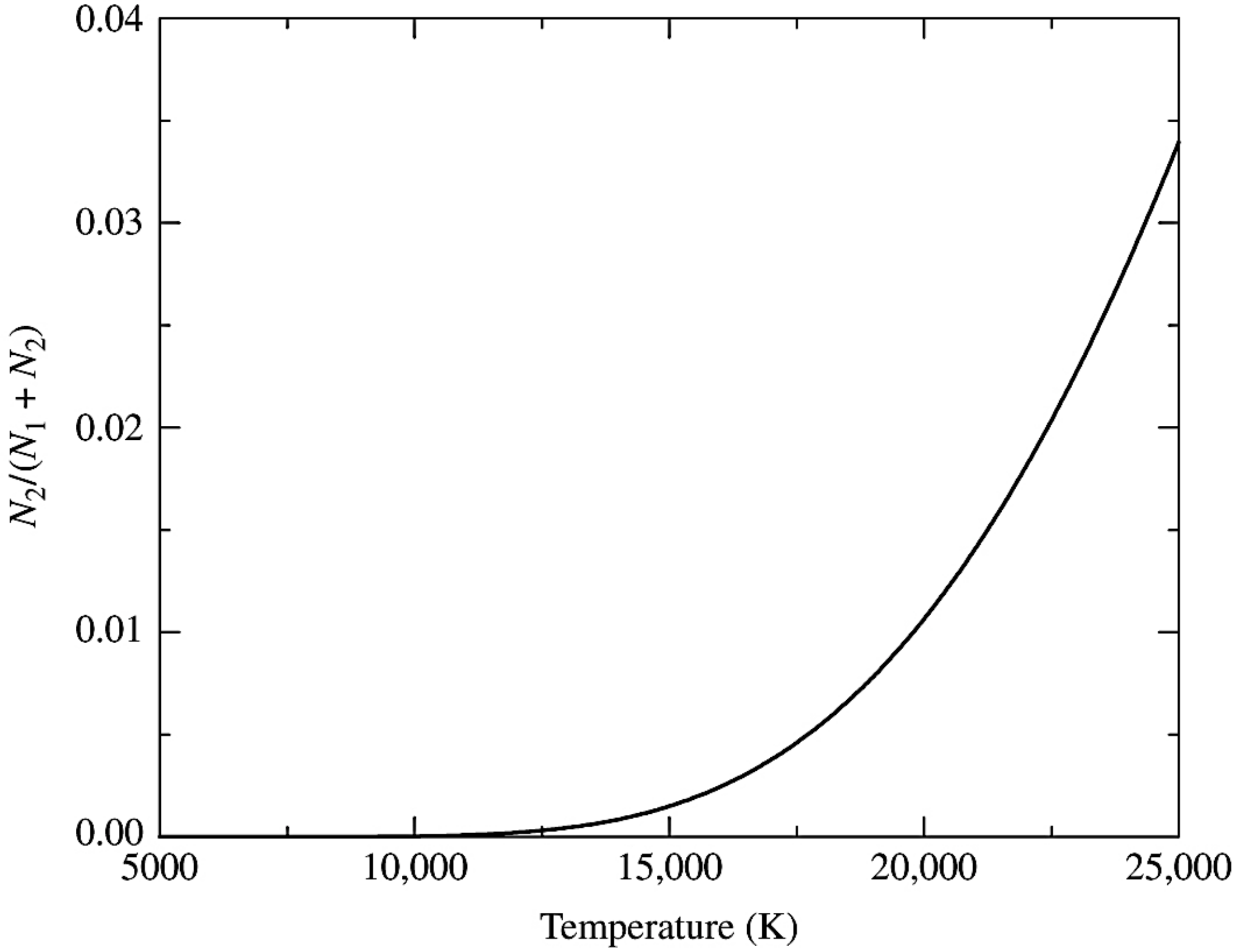
At what temperature will $N_2/N_1 = 1$?

$g_1 = 2$, $g_2 = 8$, $E_1 = -13.6$ eV, $E_2 = -13.6/2^2$ eV = -3.4 eV

$1 = (8/2)\, e^{-(-3.4\text{ eV} + 13.6\text{ eV})/kT}$

$T = 8.54\times10^4$ K

At lower temperatures $N_2/N_1 \ll 1$, why are hydrogen Balmer lines the strongest for A stars, whose effective temperatures are below 10,000 K?


0.04
0.03
0.02
0.01
0.00
$N_2/(N_1 + N_2)$
5000
10,000
15,000
20,000
25,000
Temperature (K)