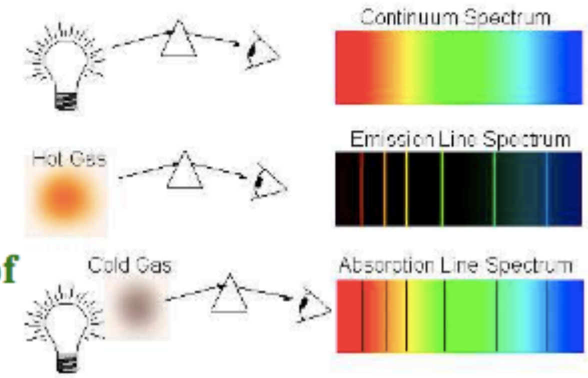
Astronomy 404

September 4, 2013

Review of last lecture:

Kirchhoff's laws of spectral analysis

1. **Hot, dense gas or solid**
   => **blackbody radiation**
2. **Hot, diffuse gas**
   => **energy level** ↓
3. **Cool, diffuse gas in front of continuum source**
   => **energy level** ↑



- $\Delta\lambda / \lambda_{rest} = v_r / c$; $E_{photon} = h\nu = hc/\lambda = pc$
- $E_n = \frac{-13.6\ Z^2 eV}{n^2}$ where Z is the charge of the nucleus

Transitions of an electron from a high level to a low level

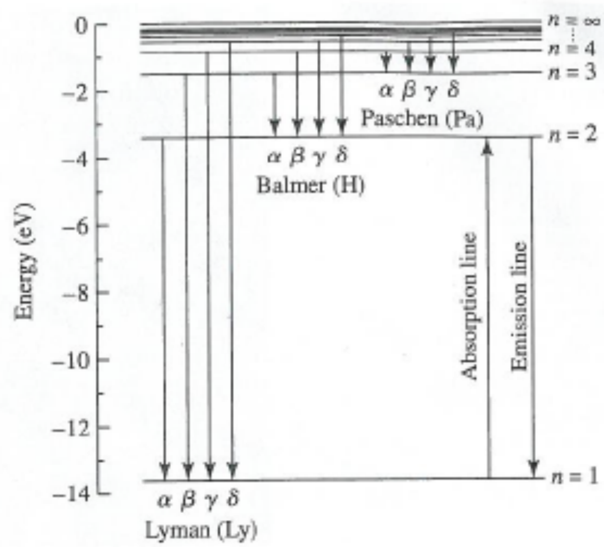


FIGURE 5.7 Energy level diagram for the hydrogen atom showing Lyman, Balmer, and Paschen lines (downward arrows indicate emission lines; upward arrow indicates absorption lines).

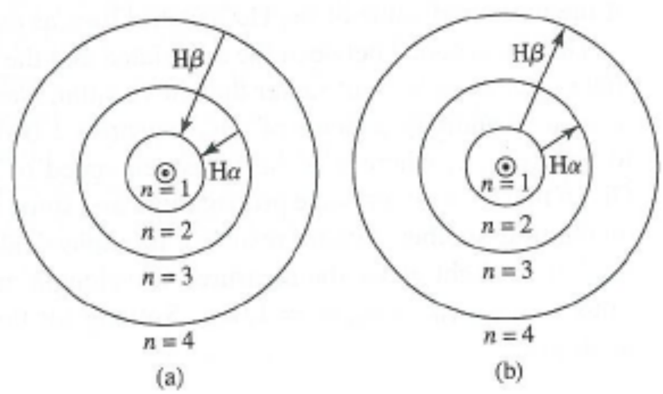


FIGURE 5.6 Balmer lines produced by the Bohr hydrogen atom. (a) Emission lines. (b) Absorption lines.

Quantum Mechanics and Wave-Particle Duality

Light can be wave, so can particles with masses, even people.

$$\nu = \frac{E}{h} \qquad \lambda = \frac{h}{p}$$

Example:
a free electron with $v_e = 3\text{x}10^6$ m/s
$\lambda = h / (m_e v_e) = 6.626\text{x}10^{-34} / (9.11\text{x}10^{-31} \text{ x } 3\text{x}10^6) = 0.24$ nm

A 70 kg person jogging at 3 m/s
$\lambda = 6.626\text{x}10^{-34} / (70 \text{ x } 3) = 3.155\text{x}10^{-36}$ m

Wave is described by a function Ψ. The square of the wave amplitude $|\Psi|^2$ describes the probability of finding a photon or an electron at the location. In a double-slit experiment, photons or electrons are never found where the waves from slits 1 and 2 have destructively interfered, that is $|\Psi_1 + \Psi_2|^2 = 0$.

Heisenberg's Uncertainty Principle

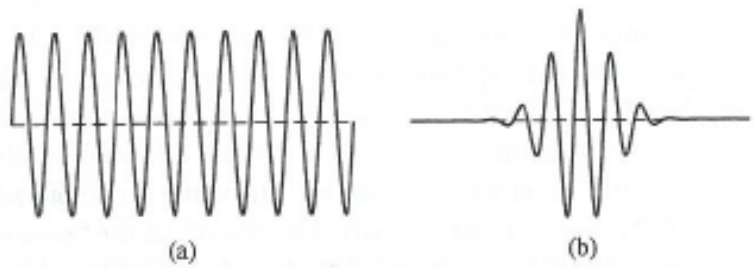


FIGURE 5.10 Two examples of a probability wave, Ψ: (a) a single sine wave and (b) composed of many sine waves.

In case (a), the wavelength is known, but the position is unknown. In case (b), a large number of waves are added together to find the location, but the momentum of the wave is not known.

This illustrates that the momentum and position cannot be determined accurately simultaneously. $\Delta x \ \Delta p \geq \hbar / 2$ Heisenberg's uncertainty principle.

$\Delta x \ \Delta p \approx \hbar$ and $\Delta E \ \Delta t \approx \hbar$ are frequently used to make order-of-magnitude estimate of physical parameters.

For example, an electron in a hydrogen atom is located in the Bohr radius a_0, so $\Delta x \approx a_0 = 5.29 \times 10^{-11}$ m.

$\Delta p = \hbar / \Delta x = 1.98 \times 10^{-24}$ kg m s^{-1}

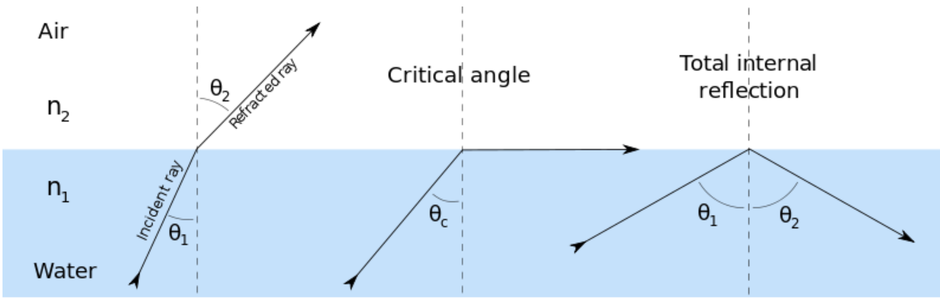
$p_{min} \approx \Delta p = m_e v_{min}$

$v_{min} = p_{min} / m_e \approx 2.18 \times 10^{6}$ m/s

$K_{min} = \frac{1}{2} m_e v_{min}^2 \approx 2.16 \times 10^{-18}$ J = 13.5 eV

$E = -K_{min} = -13.6$ eV

Quantum Mechanical Tunneling



When light enters from water to air, if the incident angle θ_1 is greater than the critical angle θc, total internal reflection occurs.

$\sin \theta_c = n_{\text{air}} / n_{\text{water}}$, where n is the index of refraction.

When total internal reflection occurs, the electromagnetic wave still goes into the air, but it ceases to be oscillatory and dies away exponentially.

In general when a classical wave enters a medium through which it cannot propagate, its amplitude decays exponentially with distance. However, if the barrier is only a few λ in thickness, wave can tunnel through and continue to be oscillatory, as shown in the figure below.

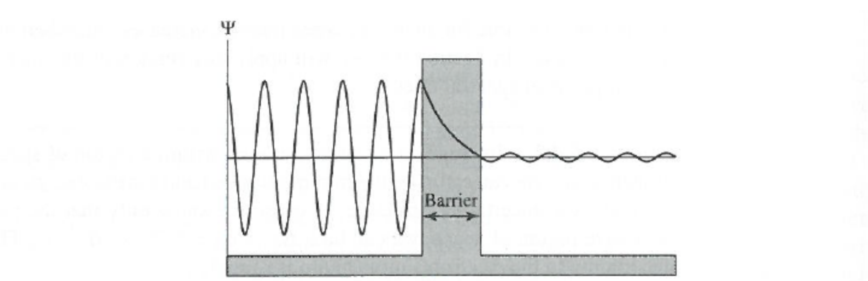


FIGURE 5.11 Quantum mechanical tunneling (barrier penetration) of a particle traveling to the right.

Tunneling (barrier penetration) is important for nuclear fusion, as thermal collisions between two nuclei cannot bring them close enough for strong force to take over.

Schrödinger's Equation and Quantum Mechanical Atom

The Schrödinger's equation is a partial differential equation that describes how the quantum state of some physical system changes with time. It can be solved for the probability waves that describe the allowed values of a particle's energy, momentum, etc.

The Schrödinger's equation can be solved analytically for the H atom.

Four quantum numbers describe the quantum state of an electron fully:

n the principal quantum number (energy)

$E_n = -13.6/n^2$ eV for H

ℓ the azimuthal quantum number (angular momentum)

$L = \sqrt{\ell(\ell+1)}\,\hbar,$ $\ell = 0, 1, 2, 3, \ldots, n-1$

m_l the magnetic quantum number (z-component of angular momentum)

$L_z = m_\ell \hbar,$ $m_l = -\ell, -\ell+1, \ldots, \ell-1, \ell$

m_s the spin quantum number (spin angular momentum)

$S_z = m_s\hbar$ $m_s = -½, +½$

Pauli's Exclusion Principle

No two electrons can have the same set of four quantum numbers.

Fermions, such as electrons, protons, and neutrons, have a spin of $\hbar/2$ (or an odd integer times $\hbar/2$), and obey Pauli's exclusion principle.

Bosons, such as photons, have an integral spin of 0, $\hbar$, $2\hbar$, $3\hbar$, …; Bosons do not obey Pauli's exclusion principle.

Selection Rules

$\Delta\ell = \pm 1$, $\Delta m_l = 0$ or ±1, but not $m_l = 0$ to $m_l' = 0$

Allowed transitions happen spontaneously on timescales of 10^{-8} s. Forbidden transitions take 1-10^8 s, depending on how many violations.