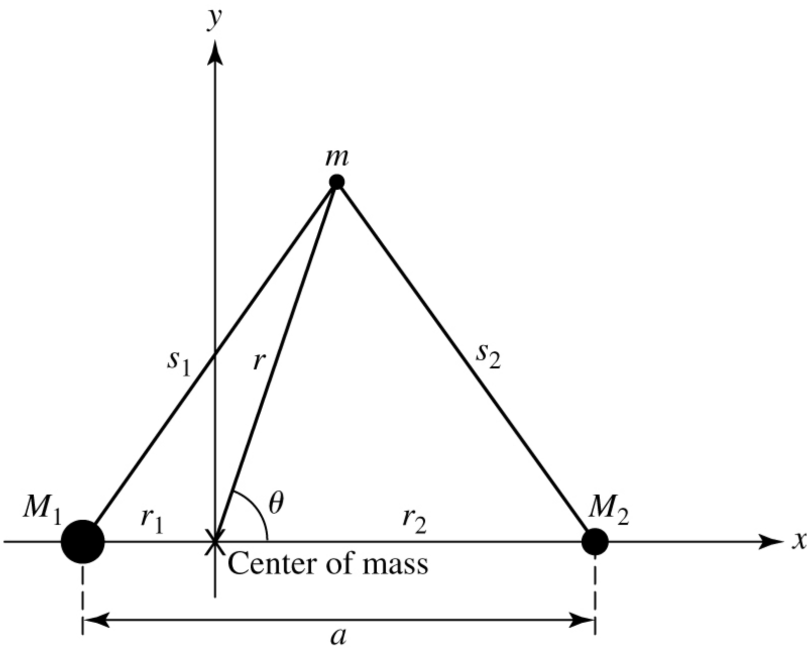
Astronomy 404
December 9, 2013

Close Binary Star Systems



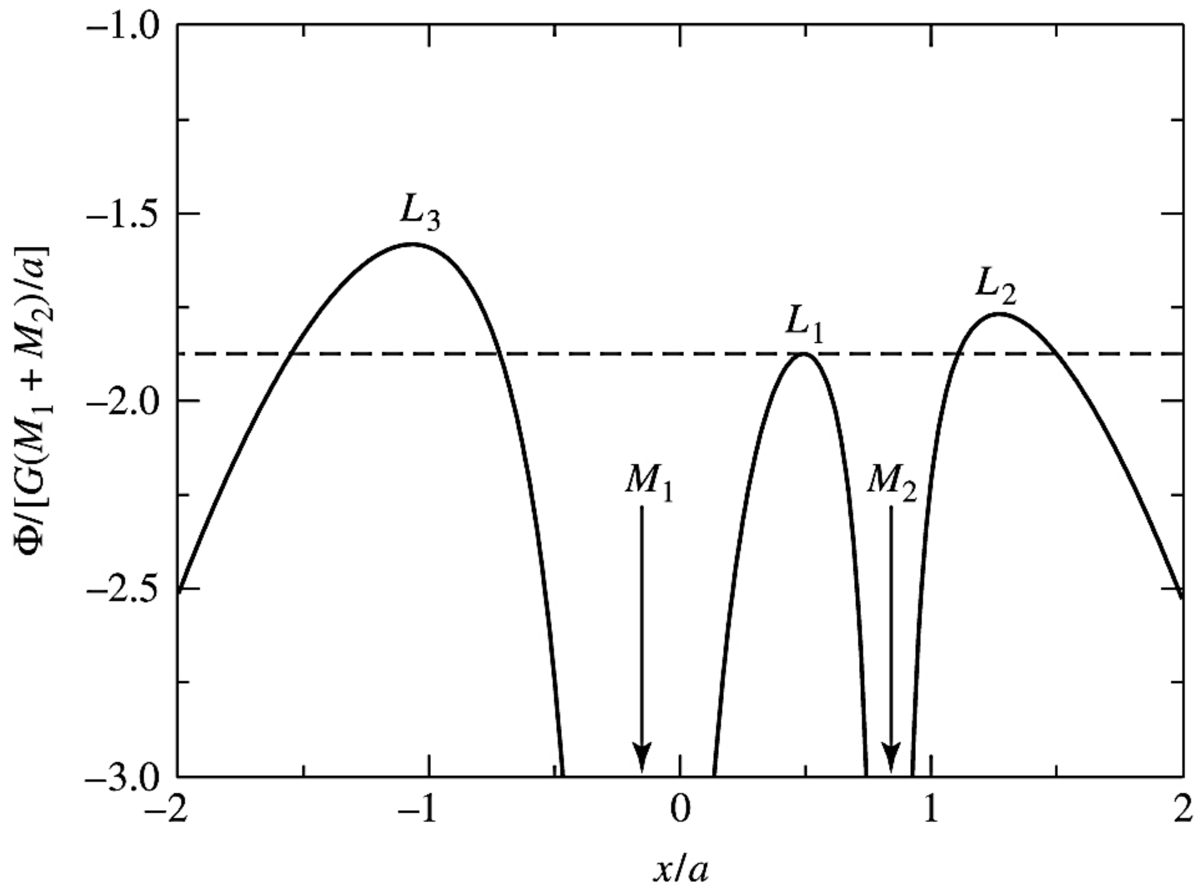
Consider two stars, M_1 and M_2, in a circular orbit in the x-y plane with an angular velocity $\omega = v_1/r_1 = v_2/r_2$.

In a rotating coordinate system, the two stars are at rest and separated by a distance a .

The effective gravitational potential is

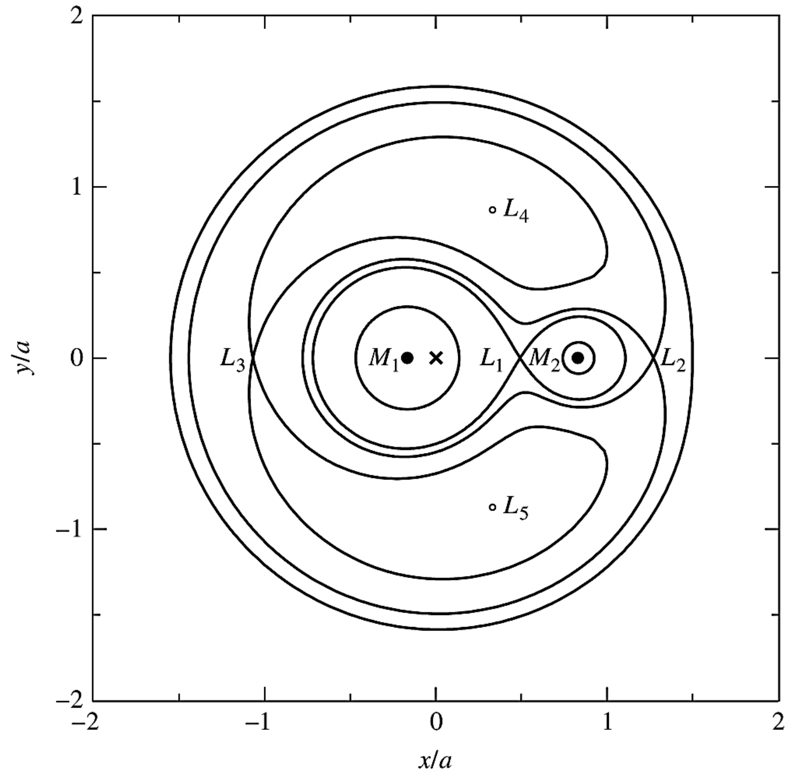
$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2.$$

$$\omega^2 = \left(\frac{2\pi}{P}\right)^2 = \frac{G(M_1 + M_2)}{a^3}$$

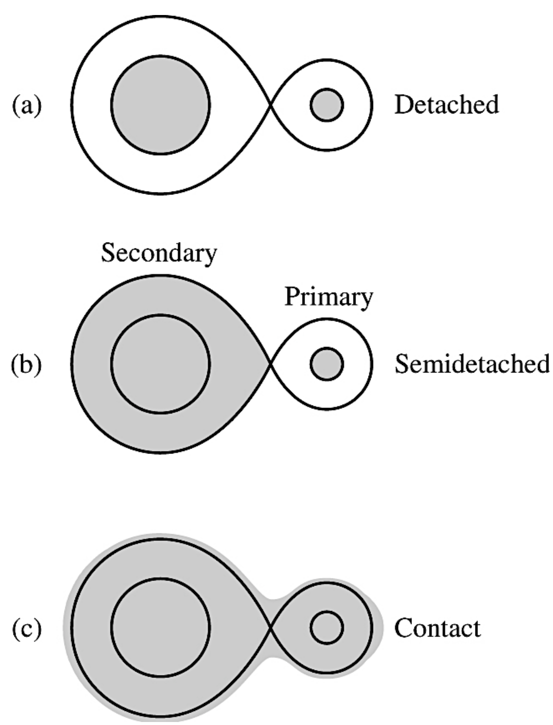


Effective potential for two stars with M_1 = 0.85 $M_\odot$ and M_2 = 0.17 $M_\odot$, separated by a = 0.718 $R_\odot$. This figure applies to any M_2/M_1 = 0.2.

The three Lagrangian points, L_1 , L_2, and L_3, are local maxima of the effective gravitational potential, where the gravitational forces are balanced by the centrifugal force. These are unstable equilibrium points.

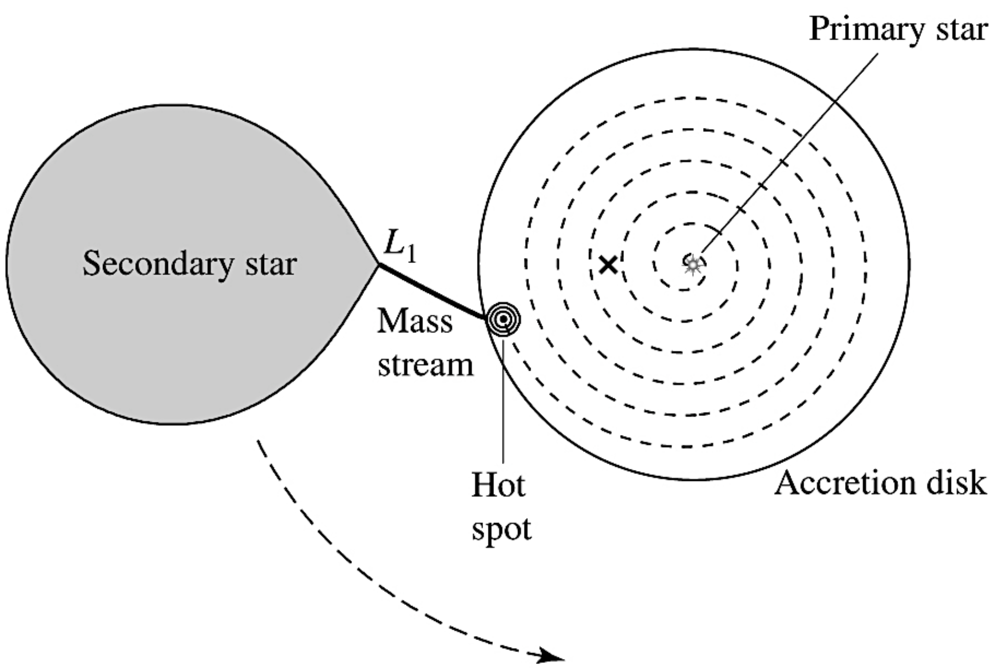


Equipotential surfaces are *level surfaces* for binary stars. The effective gravity at each point is always perpendicular to the equipotential surface.



Roche lobes. Material of the secondary star expanding beyond the Roche lobe can be acquired by the primary star - *mass transfer*.

Because of conservation of angular momentum, the mass transfer form an accretion disk orbiting around the primary star. The viscosity in the disk convert the kinetic energy into thermal energy and dissipates, causing material to spiral in gradually.



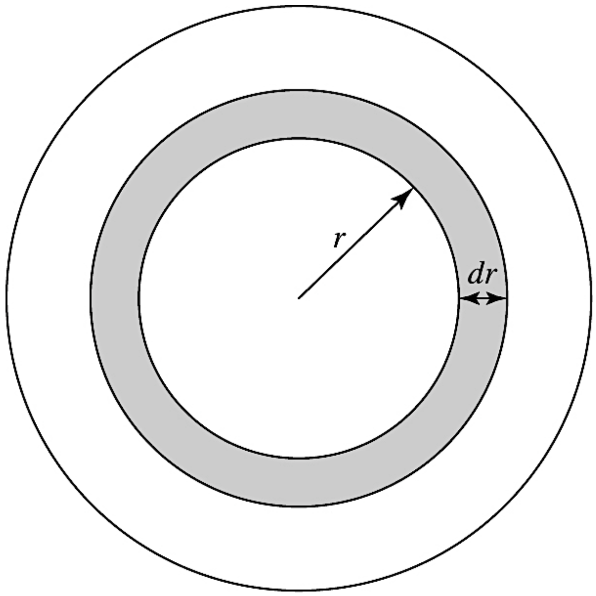
Orbit of secondary about center of mass (×)

The temperature profile of an accretion disk

The mass m orbiting around M_1 has a total energy of :

$$E = -G\frac{M_1 m}{2r}$$

Let's imagine that the accretion disk consists of annular rings.



$$dE = \frac{dE}{dr}\,dr = \frac{d}{dr}\left(-G\,\frac{M_1 m}{2r}\right)dr = G\,\frac{M_1 \dot{M} t}{2r^2}\,dr$$

where $m = \dot{M}t$.

$$dL_{\text{ring}}t = dE = G\,\frac{M_1 \dot{M} t}{2r^2}\,dr$$

The energy is radiated away as blackbody:

$$A = 2(2\pi r\,dr)$$

$$dL_{\text{ring}} = 4\pi r \sigma T^4\,dr$$

$$= G\,\frac{M_1 \dot{M}}{2r^2}\,dr$$

Solving for T, the disk temperature at radius *r* is:

$$T = \left(\frac{GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4}\left(\frac{R}{r}\right)^{3/4}$$

A more thorough analysis gives:

$$T = \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4} \left(\frac{R}{r}\right)^{3/4} \left(1 - \sqrt{R/r}\right)^{1/4}$$

$$= T_{\rm disk} \left(\frac{R}{r}\right)^{3/4} \left(1 - \sqrt{R/r}\right)^{1/4},$$

where $T_{\rm disk} \equiv \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4}$ is a characteristic temperature of the disk. It is roughly twice the maximum disk temperature:

$$T_{\rm max} = 0.488 \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4} = 0.488 T_{\rm disk}$$

At $r >> R$

$$T = \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4} \left(\frac{R}{r}\right)^{3/4} = T_{\rm disk} \left(\frac{R}{r}\right)^{3/4} \qquad (r \gg R).$$

Integrating the luminosity over radius from R to infinity,

$$L_{\rm disk} = G\frac{M\dot{M}}{2R}.$$

Without an accretion disk, the accretion luminosity is

$$L_{\rm acc} = G\frac{M\dot{M}}{R}.$$

Accretion disk temperature depends on the mass of the primary star.

Black holes and neutron stars have accretion disks in the X-ray emitting temperature range, white dwarfs' accretion disks are in the UV-emitting temperature range.