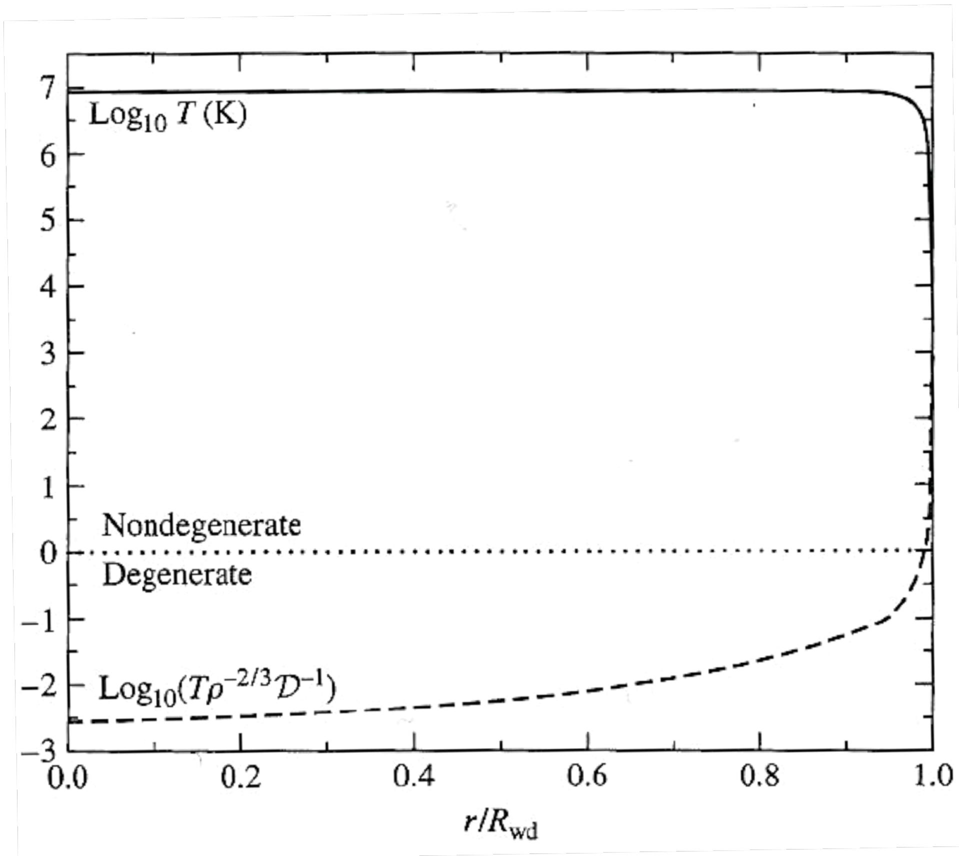
Astronomy 404
December 2, 2013

Cooling of White Dwarfs (WDs)

In the interior of a WD, energy is carried by electron conduction, which is so efficient that the interior is essentially isothermal.



The total thermal energy in the WD interior is

$$U = \frac{M_{\rm wd}}{A m_H} \frac{3}{2} k T_c$$

The luminosity of a WD is

$$L_{\rm wd} = \frac{4\mathcal{D}^3}{17}\frac{16\pi ac}{3}\frac{Gm_H}{\kappa_0 k}\mu M_{\rm wd}T_c^{7/2}$$
$$= CT_c^{7/2},$$

where

$$C \equiv \frac{4\mathcal{D}^3}{17}\frac{16\pi ac}{3}\frac{Gm_H}{\kappa_0 k}\mu M_{\rm wd}$$
$$= 6.65\times 10^{-3}\left(\frac{M_{\rm wd}}{M_\odot}\right)\frac{\mu}{Z(1+X)}$$

The cooling timescale is then:

$$\tau_{\rm cool} = \frac{U}{L_{\rm wd}} = \frac{3}{2}\frac{M_{\rm wd}k}{Am_H CT_c^{5/2}}$$

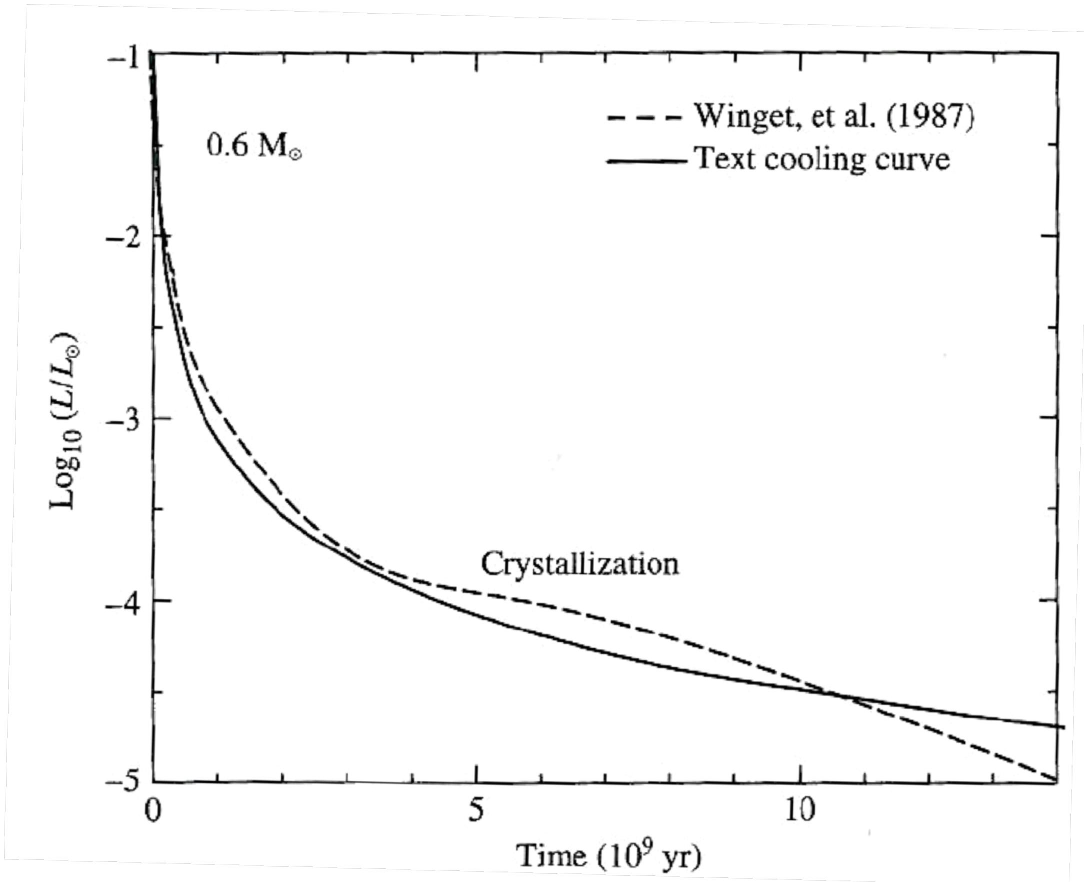
For a carbon WD, A=12, and $\tau_{1/2} = 5.2\text{x}10^{15}$ s = 170 Myr. This is an underestimate because L_{WD} decreases as the WD cools.

The change of luminosity with time:

$$-\frac{dU}{dt} = L_{\rm wd}$$
$$-\frac{d}{dt}\left(\frac{M_{\rm wd}}{Am_H}\frac{3}{2}kT_c\right) = CT_c^{7/2}$$

$T = T_0$ at $t = 0$,

$$T_c(t) = T_0\left(1+\frac{5}{3}\frac{Am_H CT_0^{5/2}}{M_{\rm wd}k}t\right)^{-2/5} = T_0\left(1+\frac{5}{2}\frac{t}{\tau_0}\right)^{-2/5}$$
$$L_{\rm wd} = L_0\left(1+\frac{5}{3}\frac{Am_H C^{2/7}L_0^{5/7}}{M_{\rm wd}k}t\right)^{-7/5} = L_0\left(1+\frac{5}{2}\frac{t}{\tau_0}\right)^{-7/5}$$



Neutron Stars

Two years after neutron was discovered by James Chadwick in 1932, Walter Baade and Fritz Zwicky proposed the existence of **neutron stars**. They also coined the term *supernova*, and suggested that supernovae represent the transitions from ordinary stars into neutron stars.

When the degenerate core of a massive star exceeds the Chandrasekhar limit (1.4 $M_\odot$), it collapses into a neutron stars. The majority of neutron stars have masses near 1.4 $M_\odot$.

Neutrons are fermions like electrons, so are subject to Pauli's exclusion principle.

A 1.4 $M_\odot$ neutron star has 10^{57} neutrons. The neutrons are held together by gravity and supported by neutron degeneracy pressure. It can be shown that

$$R_{ns} \approx \frac{(18\pi)^{2/3}}{10} \frac{\hbar^2}{G M_{ns}^{1/3}} \left(\frac{1}{m_H}\right)^{8/3}$$

For M_{ns} = 1.4 $M_\odot$, R_{ns} = 4.4 km, which is too small by a factor of ~3. Its actual radius is 10-15 km. Many uncertainties exist in the construction of a model neutron star.

The density of a neutron star, 6.7×10^{17} kg m^{-3}, is greater than the typical density of an atomic nucleus, ~2.3×10^{17} kg m^{-3}. The entire earth mass can be fit into a cue 1.5 cm on each side. The gravity on the surface of a 1.4 $M_\odot$ neutron star of 10 km radius is $g = 1.86 \times 10^{12}$ m s^{-1}, 1.9×10^{11} times the g on the Earth's surface!

Newtonian mechanics is in trouble, as

$$v_{esc} = \sqrt{2GM_{ns}/R_{ns}} = 1.93 \times 10^8 \text{ m s}^{-1} = 0.643c.$$

$$\frac{GM_{ns}m/R_{ns}}{mc^2} = 0.207$$

General theory of relativity is needed!

The Equation of State

Initially at low densities, the nucleons are found in iron nuclei. When the density is high enough and the electrons become relativistic, protons in iron nuclei can capture energetic electrons to become neutrons.

$$p^+ + e^- \rightarrow n + \nu_e$$

But the neutron mass is greater than the sum of proton and electron masses, and neutrino's rest-mass energy is negligible, so the electrons must supply the kinetic energy to make up the difference in energy:

$$m_n c^2 - m_p c^2 - m_e c^2 = 0.78 \text{ MeV}$$

Densities must exceed 10^{12} kg m^{-3} for the protons in Fe to capture electrons. At still higher densities, the electron capture will produce increasingly neutron-rich nuclei, such as

$${}^{56}_{26}\text{Fe},\ {}^{62}_{28}\text{Ni},\ {}^{64}_{28}\text{Ni},\ {}^{66}_{28}\text{Ni},\ {}^{86}_{36}\text{Kr},\ \ldots,\ {}^{118}_{36}\text{Kr}.$$

This process is called **neutronization**.

Ordinarily, these supernumerary neutrons will revert to protons via beta decay:

$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$

but under the conditions of complete electron degeneracy, there is no vacant state for an emitted electron to occupy, so the beta decay cannot happen.

When the density reaches 4×10^{14} kg m^{-3}, the minimum-energy arrangement requires some neutrons to be *outside* of the nuclei – called **neutron drip.** This marks the start of 3-component mixture of a lattice of neutron-rich nuclei, nonrelativistic degenerate n, and relativisitic degenerate e^-.

The fluid of free protons has no viscosity, because of a spontaneous pairing of the degenerate neutrons. The resulting combination of two fermions (neutrons) is a boson and is not subject to Pauli's exclusion principle. Degenerate bosons can *all* crowd into the lowest energy state, and the fluid of paired neutrons can lose no energy. It is a **superfluid** that flows without resistance.

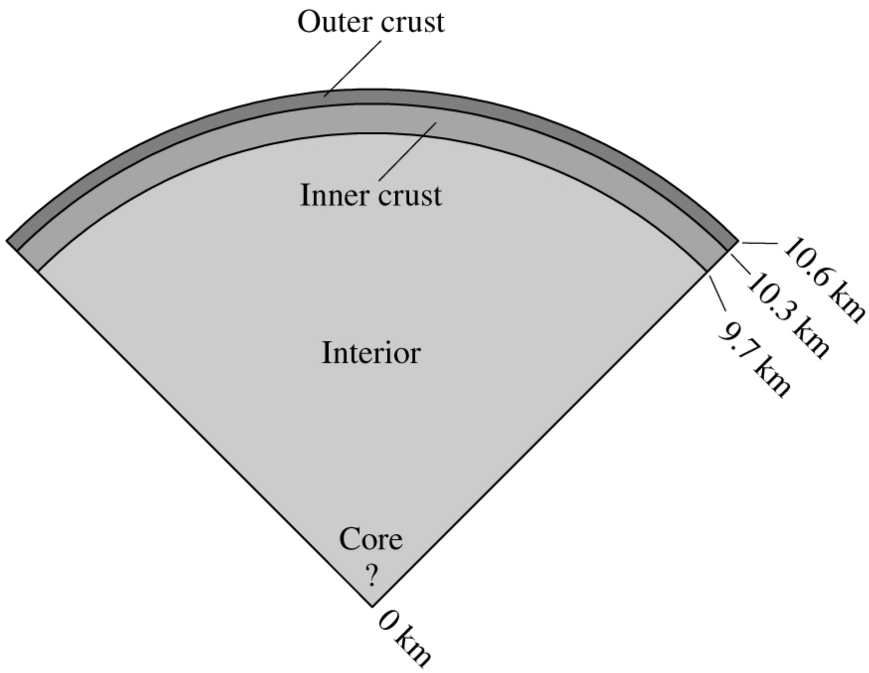
As the density increases still higher, $4x10^{15}$ kg m^{-3}, there is effectively no distinction between inside and outside of nuclei. The fluid becomes **superconducting**, with zero electric resistance.

The properties of the neutron star material at very high densities are still poorly understood. A complete theoretical description of the behavior of a sea of free neutrons interacting via the strong nuclear force in the presence of protons and electrons is not yet available.

Compositions of Neutron Star Material:

Transition density $(kg\ m^{-3})$	Composition	Degeneracy pressure
	iron nuclei, nonrelativistic free electrons	electron
$\approx 1 \times 10^{9}$	electrons become relativistic	
	iron nuclei, relativistic free electrons	electron
$\approx 1 \times 10^{12}$	neutronization	
	neutron-rich nuclei, relativistic free electrons	electron
$\approx 4 \times 10^{14}$	neutron drip	
	neutron-rich nuclei, free neutrons, relativistic free electrons	electron
$\approx 4 \times 10^{15}$	neutron degeneracy pressure dominates	
	neutron-rich nuclei, superfluid free neutrons, relativistic free electrons	neutron
$\approx 2 \times 10^{17}$	nuclei dissolve	
	superfluid free neutrons, superconducting free protons, relativistic free electrons	neutron
$\approx 4 \times 10^{17}$	pion production	
	superfluid free neutrons, superconducting free protons, relativistic free electrons, other elementary particles (pions, …?)	neutron

A 1.4 solar mass neutron star model:



(1) Outer crust – heavy nuclei, in the form of either a fluid ocean or a solid lattice

(2) Inner crust – three part mixture of a lattics of nuclei, a superfluid of free neutrons, and relativistic electrons.

(3) Interior – superfluid neutrons

(4) Solid core – consisting of pions and sub-nuclear particles; may or may not exist…

Neutron stars have rapid rotation (conservation of angular momentum) and strong magnetic field (freezing in mag field lines). If the dipole magnetic field is inclined to the rotation axis, there will be magnetic dipole radiation – a pulsar.