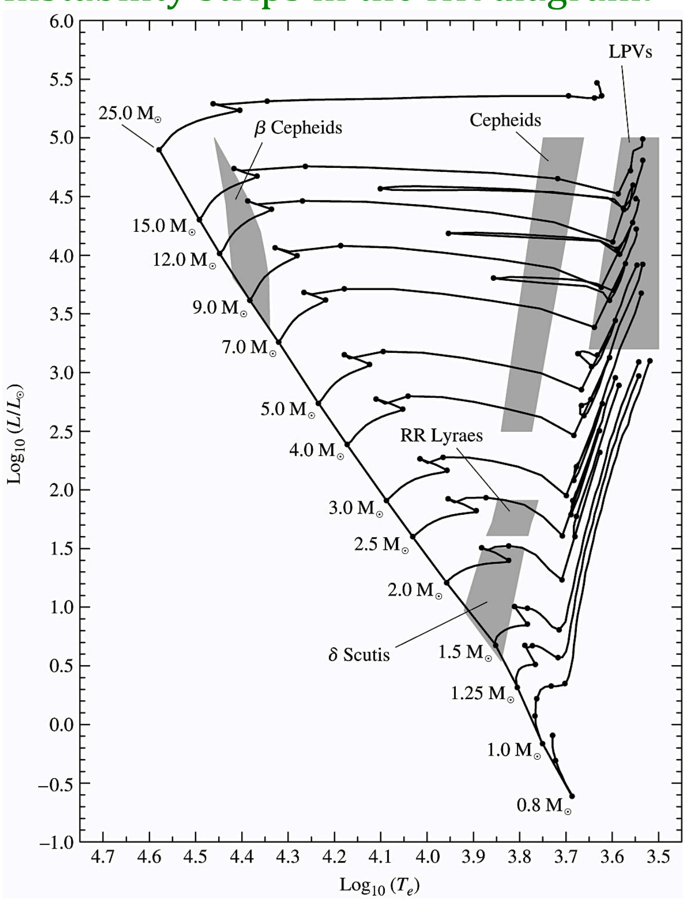
Astronomy 404
November 13, 2013

Chapter 14. Stellar Pulsation

Classical Cepheids' period-luminosity relation can be used to determine distances.

$$M_{\langle V \rangle} = -2.81 \log_{10} P_{\mathrm{d}} - 1.43,$$

Instability strips in the HR diagram:



Type	Range of Periods
Long-Period Variables	100–700 days
Classical Cepheids	1–50 days
W Virginis stars	2–45 days
RR Lyrae stars	1.5–24 hours
δ Scuti stars	1–3 hours
β Cephei stars	3–7 hours
ZZ Ceti stars	100–1000 sec

The pulsation periods decrease down the instability strip. Why?

The radial oscillations of a pulsating star are the result of **sound waves resonating in the star's interior.**

The adiabatic sound speed is $v_s = (\gamma P/\rho)^{1/2}$, where $\gamma \equiv \frac{C_P}{C_V}$.
The pulsation period can be estimated from the hydrostatic equilibrium equation with a simplistic (unrealistic) assumption of uniform density:

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\rho}{r^2} = -\frac{4}{3}\pi G\rho^2 r$$

Using the boundary condition of $P = 0$ at the surface, we get

$$P(r) = \frac{2}{3}\pi G\rho^2\left(R^2 - r^2\right)$$

The pulsation period is roughly

$$\Pi \approx 2\int_0^R \frac{dr}{v_s} \approx 2\int_0^R \frac{dr}{\sqrt{\frac{2}{3}\gamma\pi G\rho\left(R^2 - r^2\right)}}$$

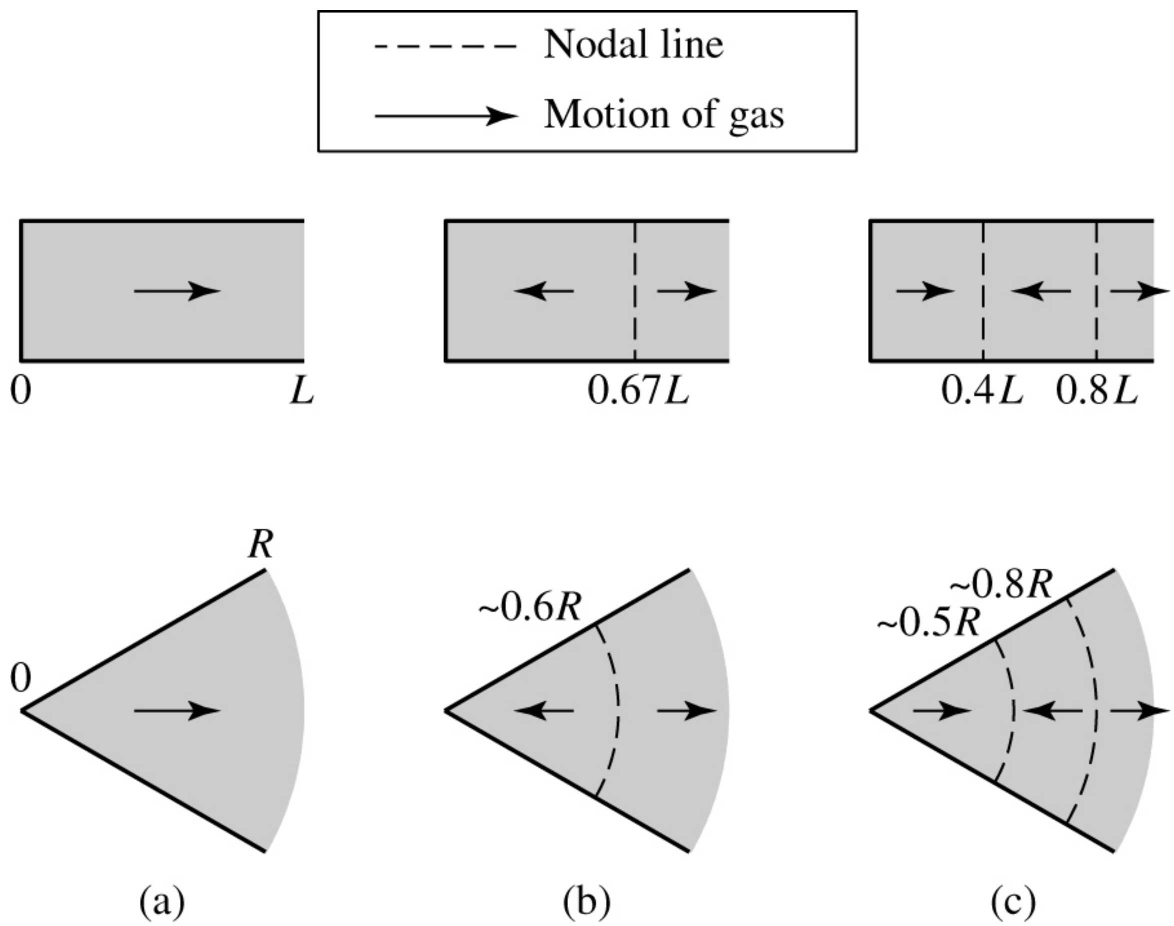
or the **period-mean density relation**.

$$\boxed{\Pi \approx \sqrt{\frac{3\pi}{2\gamma G\rho}}.}$$

If we take $M = 5\ M_\odot$ and $R = 50\ R_\odot$ for a typical classical Cepheid, then $\Pi \approx 10$ days, right in the observed range of periods.

The classical Cepheids are in the instability strip with a small temperature range, and thus follow a period-luminosity relation.

In the radial modes of stellar pulsation, the sound waves are essentially *standing waves*, similar to those in an organ pipe with one end open.

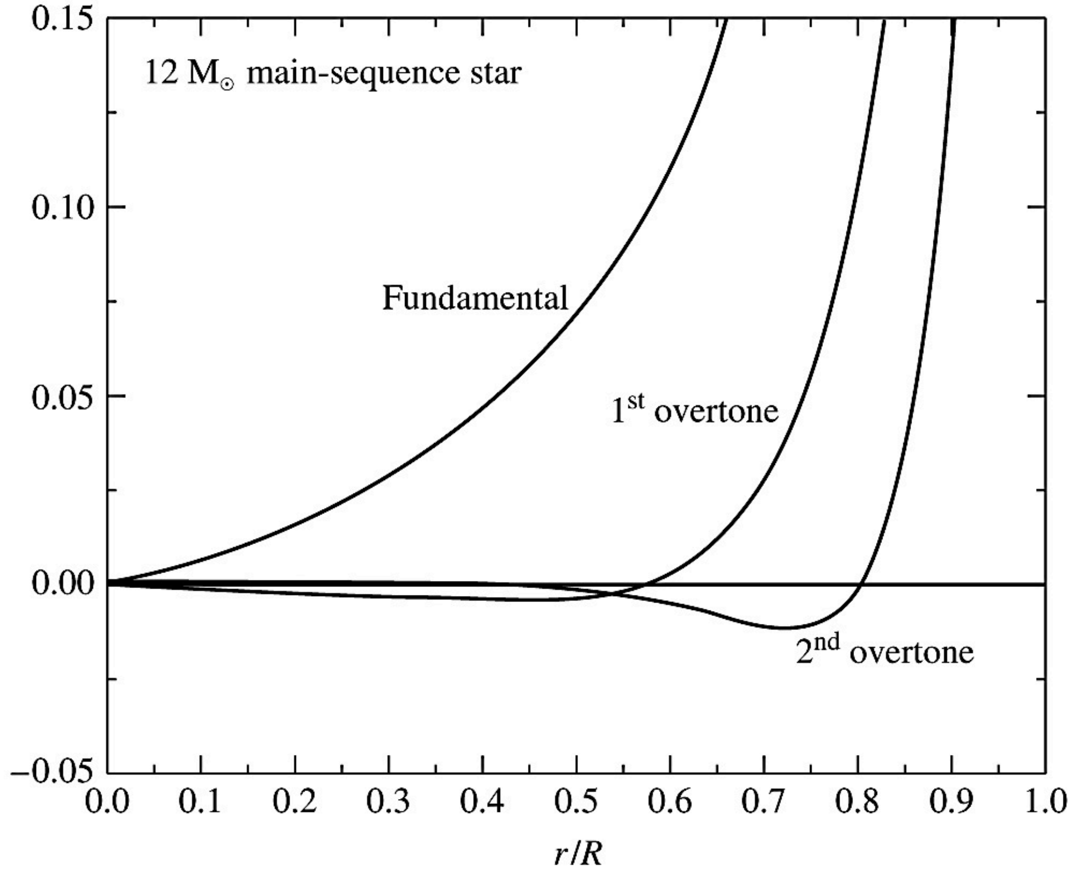


(a) fundamental mode – gases move in the same direction

(b) first overtone, or first harmonic – a single node where gases move in opposite directions

(c) second overtone, or second harmonic – two nodes where gases move in opposite directions.

The fractional displacement of the stellar material from its equilibrium position for a 12 $M_\odot$ main sequence star model:



The amplitude is the largest for the fundamental mode. The displacement is larger toward the stellar surface.

The vast majority of classical Cepheids and W Virginis variables (Cepheid II) pulsate in the fundamental mode.

RR Lyrae variables pulsate in either the fundamental or first overtone mode, with some oscillating in both modes simultaneously.

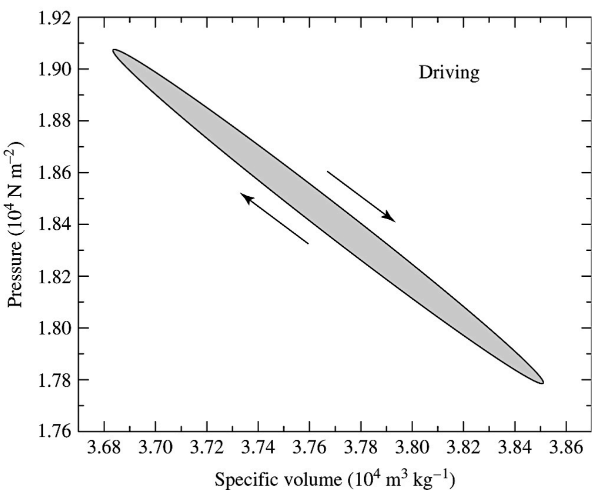
LPVs may oscillate in either the fundamental mode or the first overtone.

What powers these standing sound waves?

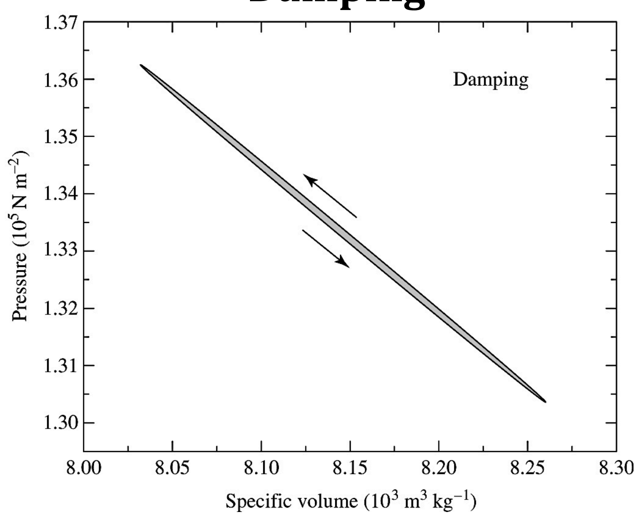
Eddington's Thermodynamic Heat Engine

Eddington proposed that pulsating stars are heat engines. The gases comprising the layers of the star do *PdV* work as they contract and expand throughout the pulsation cycle. If $\oint P\,dV > 0$ for the cycle, a layer does net positive work and drives the oscillation. If $\oint P\,dV < 0$ for the cycle, the layer does net negative work and damps the oscillation.

Driving



Damping



For driving, the heat must enter the layer during the high-temperature part of the cycle and leave during the low- temperature part. The driving layers of a pulsating star must absorb heat around the time of their maximum compression. In this case, the maximum pressure will occur after the maximum compression.

The Nuclear ε Mechanism

When the center of the star is compressed, the temperature and density rise, and the nuclear energy generation rate increases, driving a stellar pulsation, but the amplitude at the stellar core is too small to affect the entire star. This mechanism is proposed to prevent the formation of massive stars with $M > 90\ M_\odot$.

Eddington's Valve

Eddington suggested an alternative, a *valve mechanism*. If a layer of the star becomes more opaque during compression, it could "dam up" the energy flowing toward the surface and push the surface layer upward. As the expanding layer becomes ore transparent, the trapped heat escapes and the layer falls back down to begin the cycle anew. *The opacity must increase when compressed.*

Opacity can be described by Kramer's law $\kappa \propto \rho/T^{3.5}$. When stellar material is compressed, the opacity usually decreases because κ's sensitivity to temperature.

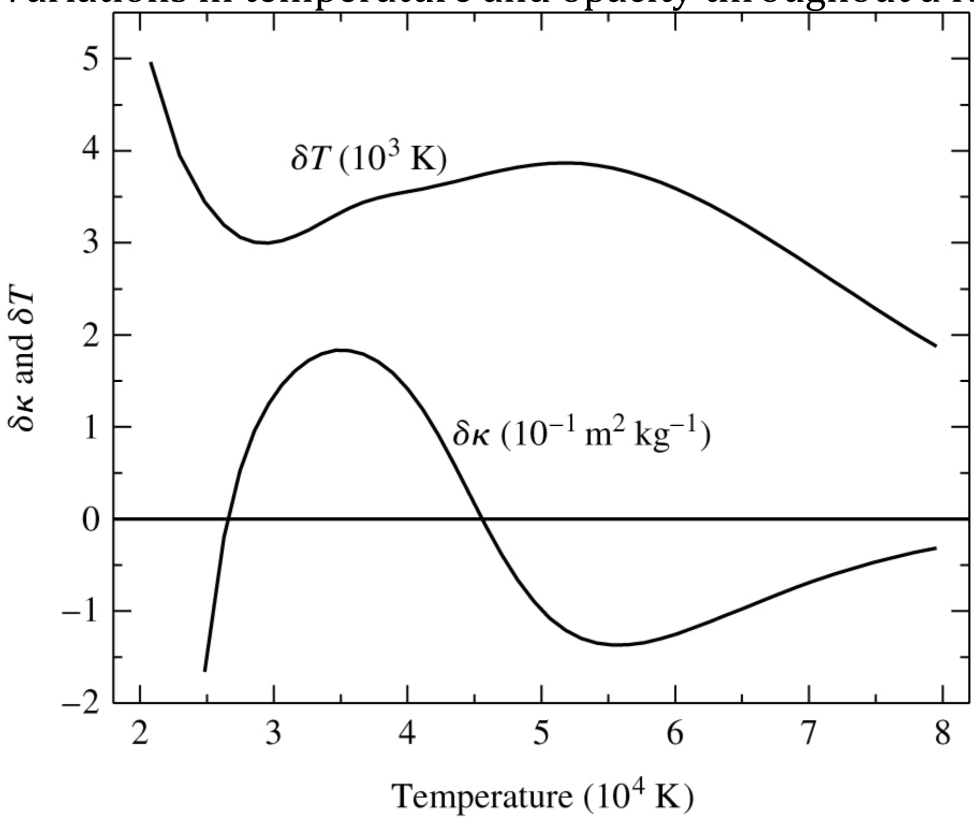
Opacity Effects and the κ and γ Mechanisms

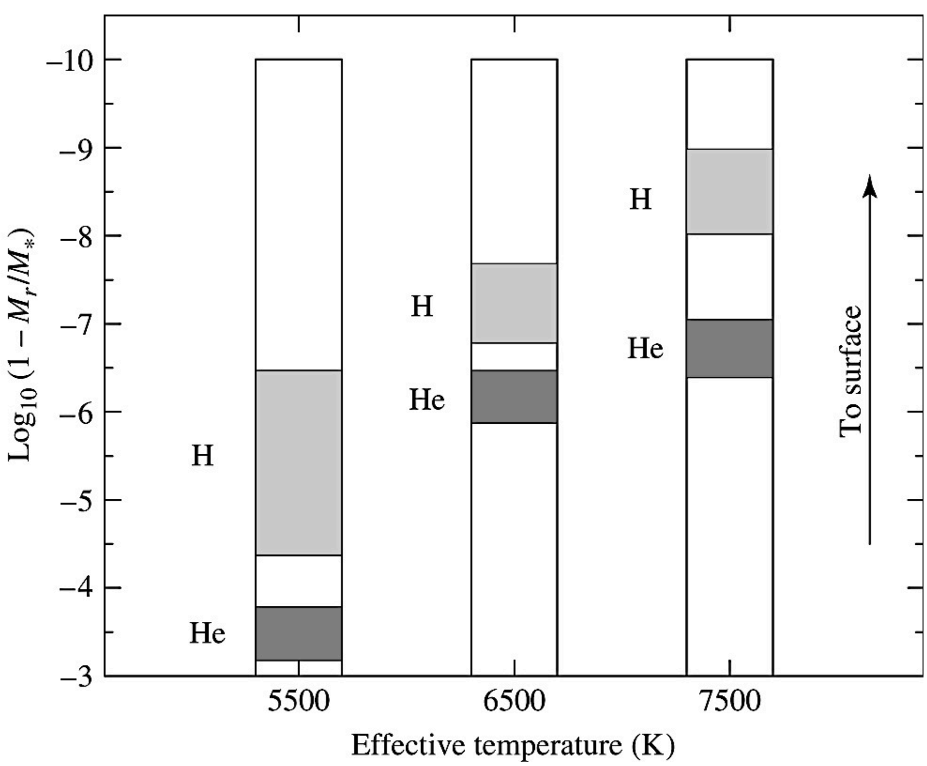
In partial ionization zones of a star, when the gas is compressed part of the work done on the gas produce further ionization, rather than raising the temperature. The compression raises the density a lot, but not so much in temperature, causing a net increase in κ. As the layer expands, the density decreases and the opacity decreases. So, the partial ionization layer absorbs heat during compression, pushes outward to release heat during expansion, and falls back down again to start another cycle. This is called **κ-mechanism.**

The κ-mechanism is reinforced by the changes of specific heat in C_P and C_V, resulting in a smaller γ. This effect is called **γ-mechanism**.

The partial ionization zone of H I ➔ H II occurs at 1×10^4 K.
He I ➔ He II occurs at 1.5×10^4 K
He II ➔ He III occurs at 4×10^4 K.

Variations in temperature and opacity throughout a RR Lyrae star.





If a star is too hot (7500 K), the partial ionization zone is too close to the surface and does not have enough mass to drive the oscillations effectively.   This causes the **blue edge** of the instability strip.

In the cooler stars (6500 K), the ionization zones are deeper in the star, so more mass is in the "piston" and even the first overtone mode can be excited.

In a still cooler star (5500 K), the ionization zones occur deep enough to excite the fundamental mode of pulsation.

At still cooler temperatures, the convection in the outer layer of the star can dampen the oscillation, as convection helps lose energy during the maximum compression, overcoming the damming up of heat by the ionization zones. This causes the **red edge** of the instability strip.

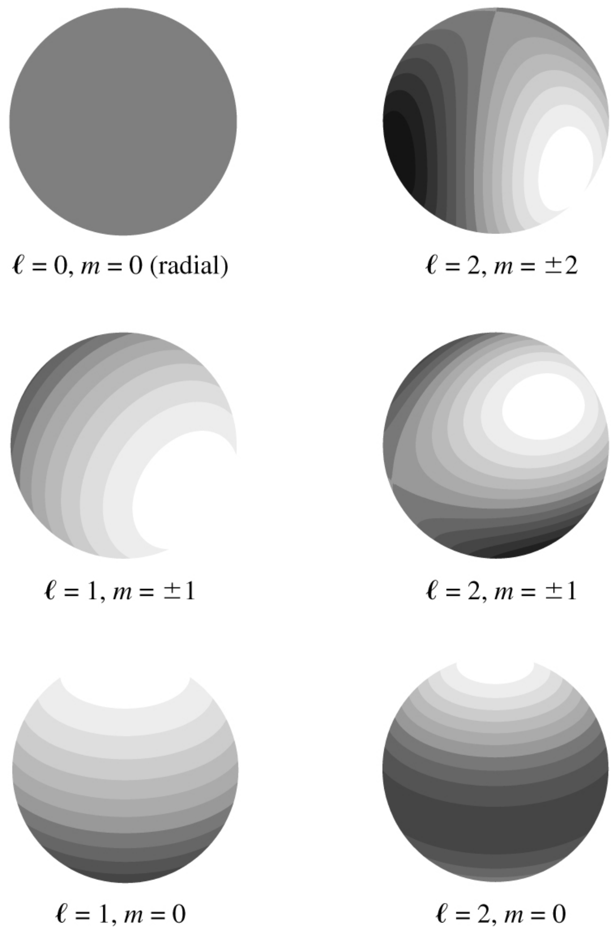
Β Cephei stars pulsate via κ- and γ-mechanisms, but the driving element is Fe. Pulsating stars outside the instability strips are not well understood.

To model the stellar pulsation, we start with Newton's second law

$$\rho \frac{d^2 r}{dt^2} = -G \frac{M_r \rho}{r^2} - \frac{dP}{dr}.$$

This nonlinear equation can be linearized and solved numerically.

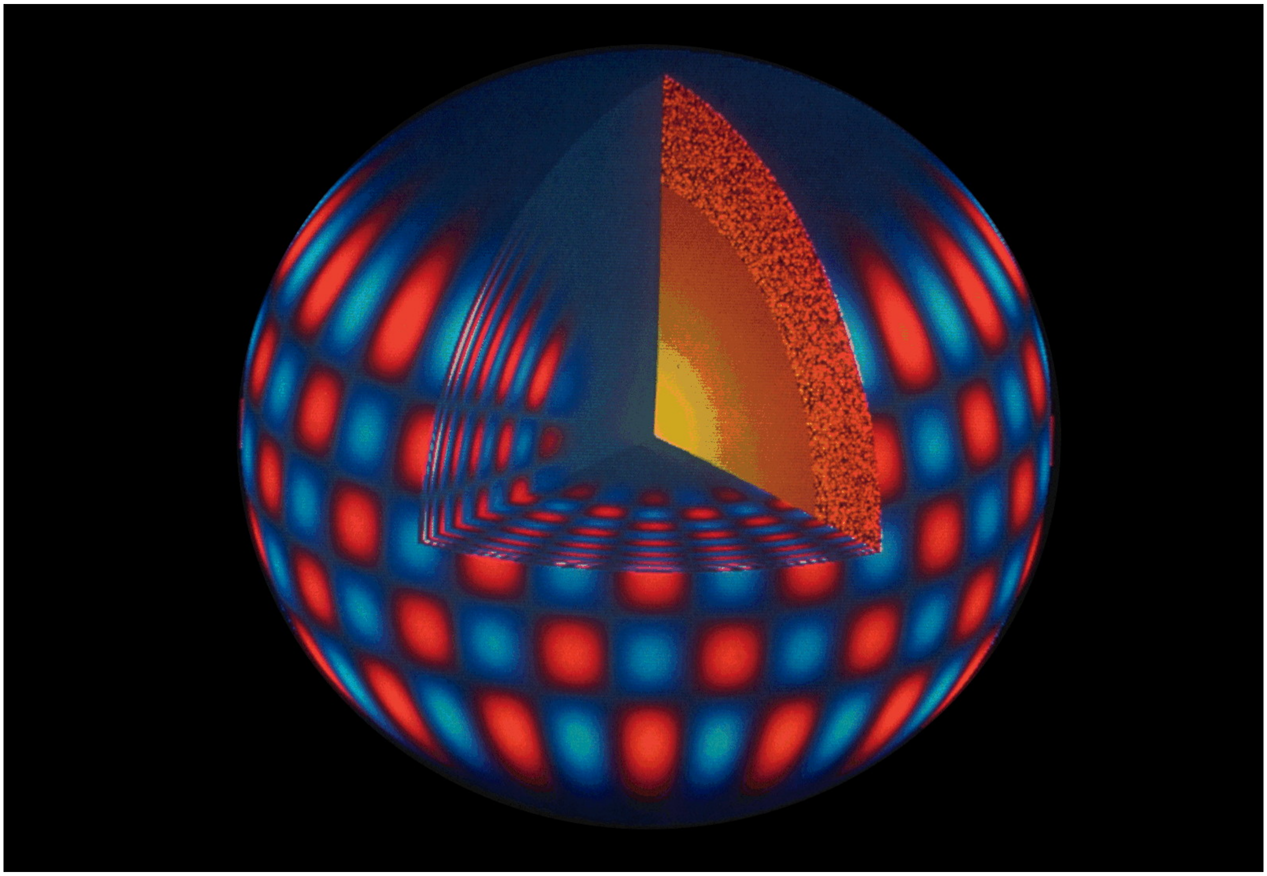
Some stars have non-radial pulsation.

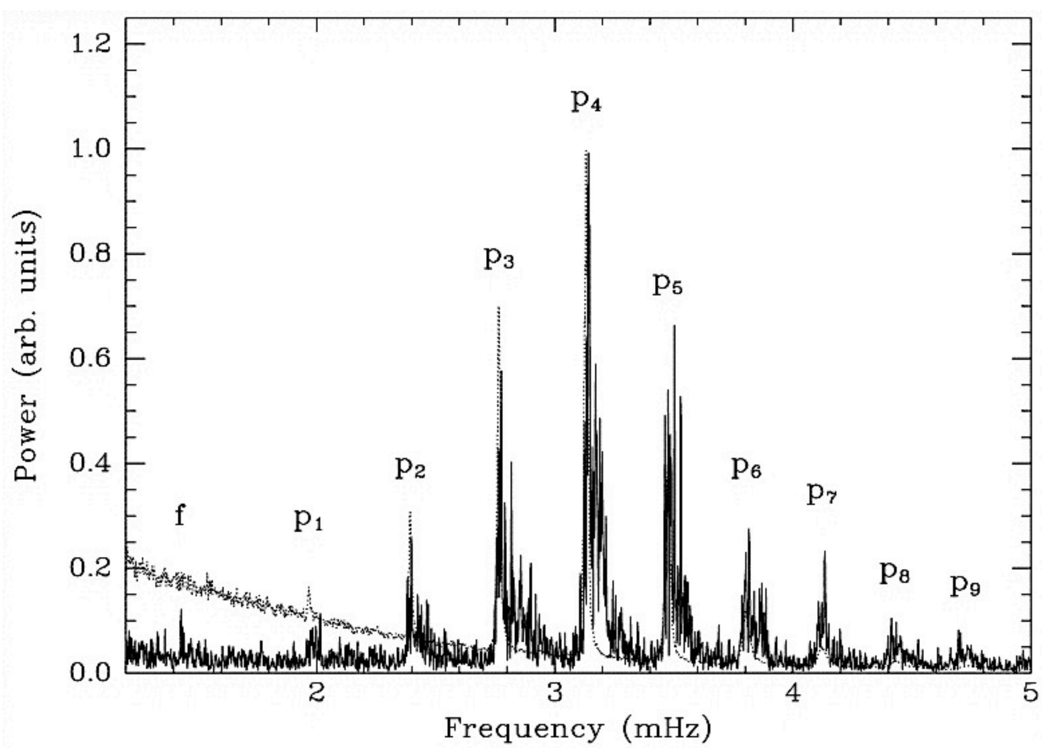


p mode – nonradial oscillation, pressure provides restoring force
f mode – surface gravity wave
g mode – produced by internal gravity waves, “sloshing”

The g and p modes of stellar oscillation can be used to probe stellar structure.

The g mode of the Sun has not been definitively identified, but the five-minute p-mode oscillation is well studied.   It’s from the convection zone.





1.2
1.0
0.8
0.6
0.4
0.2
0.0
Power (arb. units)
f
p_1
p_2
p_3
p_4
p_5
p_6
p_7
p_8
p_9
2
3
4
5
Frequency (mHz)