Astronomy 404
October 30, 2013

Chapter 13. Main Sequence and Post-Main Sequence Stellar Evolution

Timescales to pay attention to:

Free-fall timescale	-- pre-main sequence	(~0.4 Myr)
Kelvin-Helmholtz timescale	-- pre-main sequence	(10 Myr for Sun)
Nuclear timescale	-- main sequence	(10 Gyr for Sun)

The nuclear timescale is the longest, during which a star is stable and is located in the main sequence in an H-R diagram.

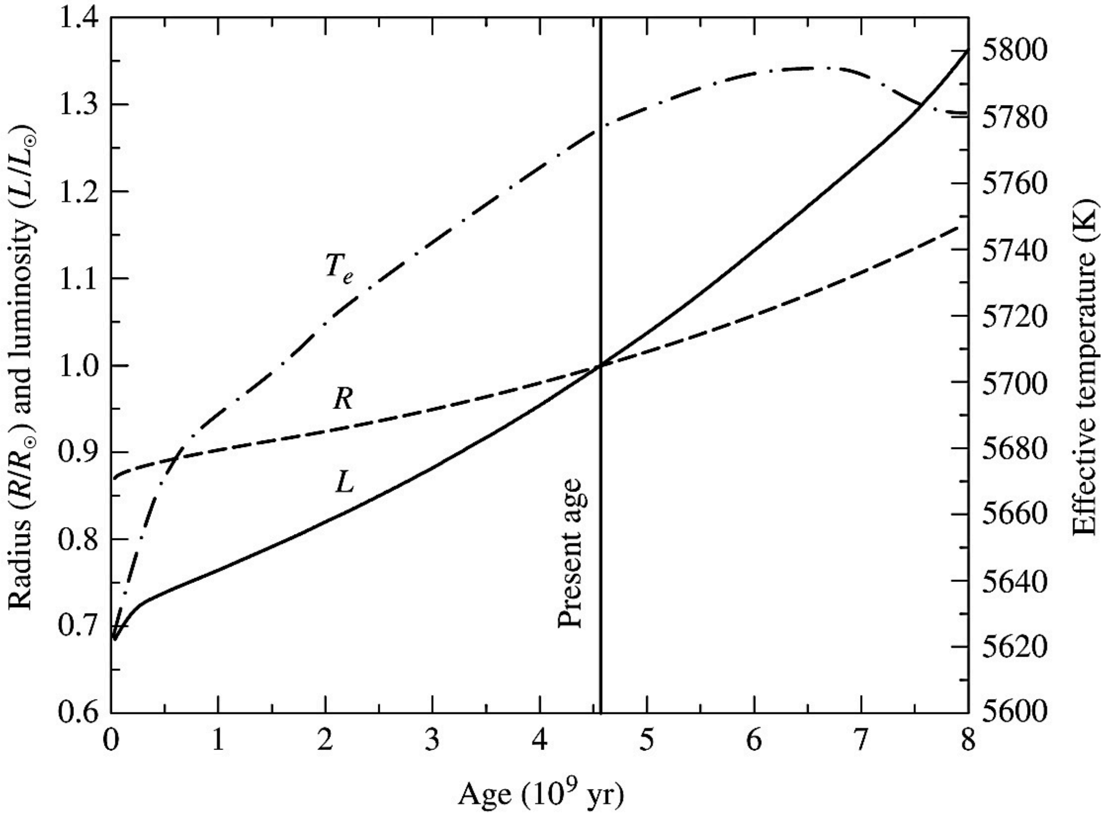
The main sequence is not a thin line; its widths are caused by the stars adjusting their internal structure as nuclear reactions go on.

Remember that H-burning in stars more massive than 1.2 $M_{\odot}$ is dominated by CNO cycle and the lower mass stars PP chain. These different reactions have different energy production rates, which lead to different internal structures and different evolution paths.

Stellar evolution is a saga of stars fighting the gravitational force!

For a low-mass main sequence star like the Sun,
PP chain converts H into He in the stellar core => μ increases
As $P = (\rho/\mu\, m_H)\, k\, T$, ρ and/or T need to increase to maintain the pressure. Therefore, the core must be compressed.

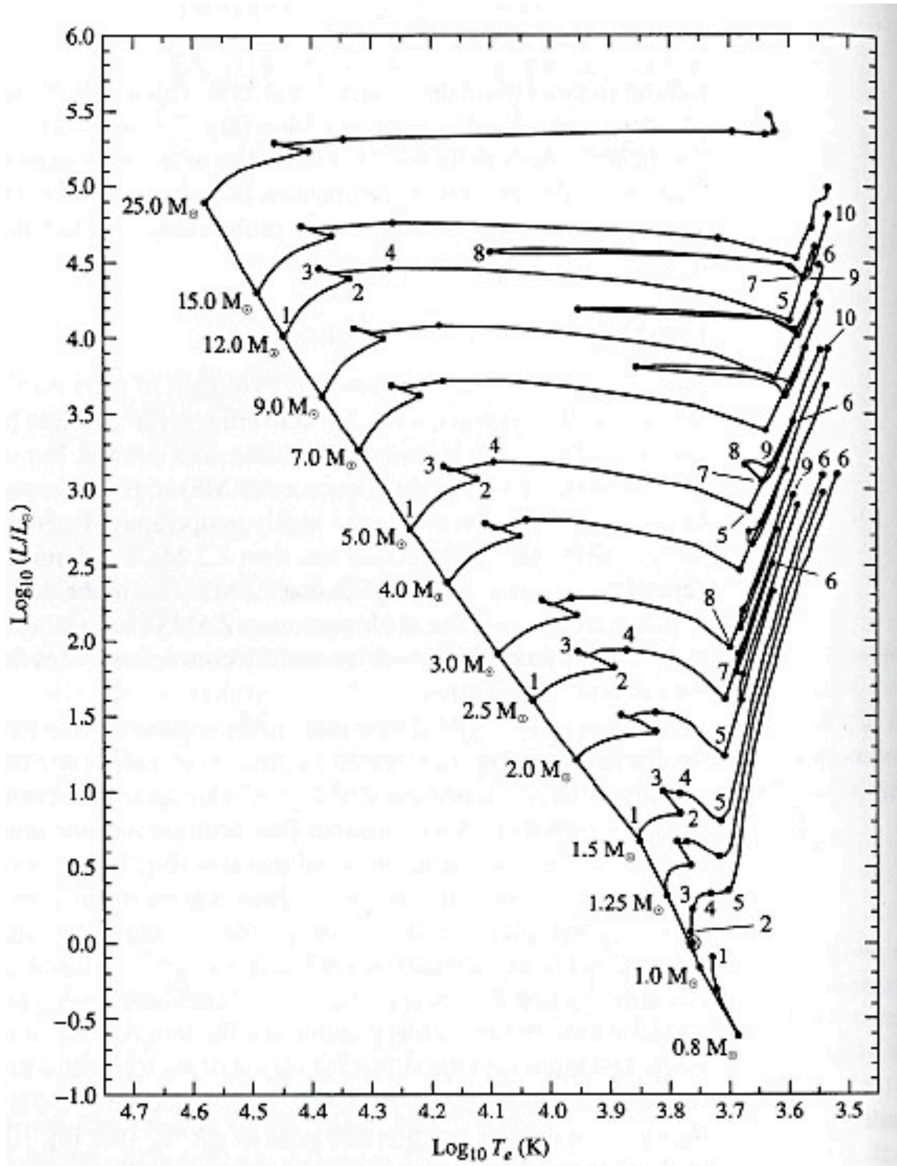
The compressed core raises the density, and ½ of the gravitational energy released is radiated away and ½ is retained to heat the core. The raised density and temperature in the core raises the nuclear reaction rate and energy production rate, although the fraction of H is lower than before. (reaction rate $\propto \rho X^2 T^4$)
Therefore, stellar luminosity slowly increases.



For the Sun, luminosity, radius, and temperature all increases.

For stars of different masses, the evolutionary tracks are different.

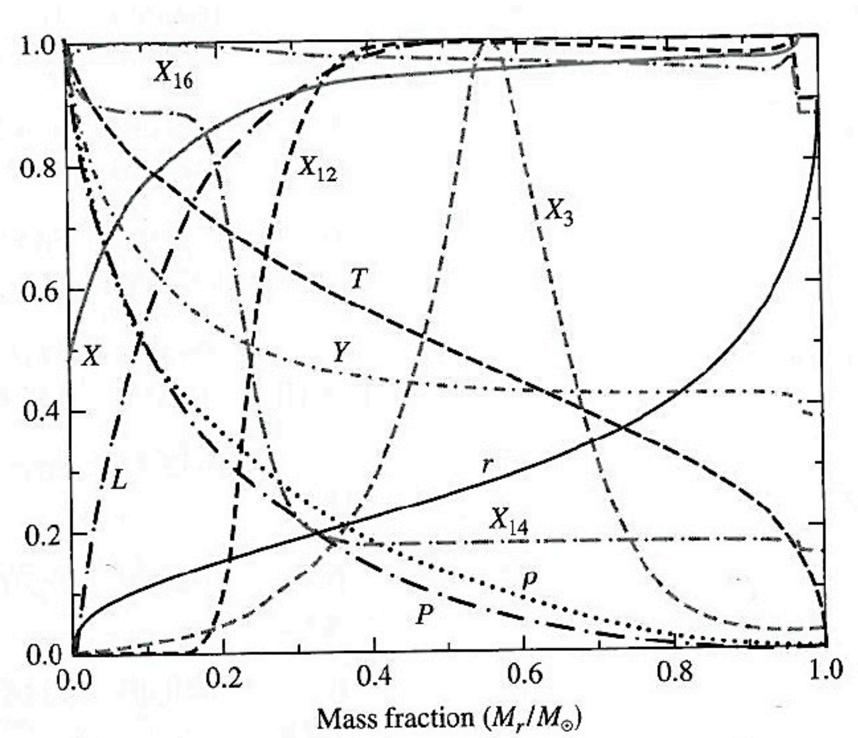
The first evolutionary tracks of low-mass stars were calculated by Professor Icko Iben, Jr., an emeritus professor of our department.



After core H is exhausted, H-burning occurs in a shell around the He core. The core (with 3% of the mass) has zero luminosity and is isothermal.

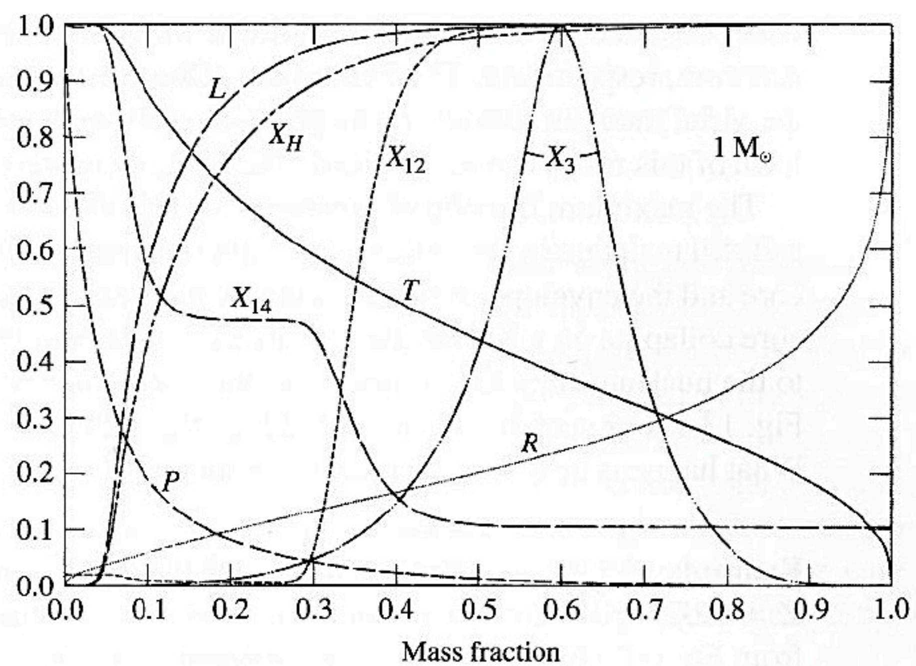
$L_r = 0$, so $dT/dr = 0$ => constant T in the core.

What determined the evolutionary tracks?
Contraction or expansion of the core, contraction or expansion of the envelope, subsequent changes in energy generation rates…



Above is the present-day Sun's structure.

Below is the Sun's structure near point 3 of the evolutionary track.



Initial Mass ($M_\odot$)	1 6	2 7	3 8	4 9	5 10
25	0	6.33044	6.40774	6.41337	6.43767
	6.51783	7.04971	7.0591		
15	0	11.4099	11.5842	11.5986	11.6118
	11.6135	11.6991	12.7554		
12	0	15.7149	16.0176	16.0337	16.0555
	16.1150	16.4230	16.7120	17.5847	17.6749
9	0	25.9376	26.3886	26.4198	26.4580
	26.5019	27.6446	28.1330	28.9618	29.2294
7	0	42.4607	43.1880	43.2291	43.3388
	43.4304	45.3175	46.1810	47.9727	48.3916
5	0	92.9357	94.4591	94.5735	94.9218
	95.2108	99.3835	100.888	107.208	108.454
4	0	162.043	164.734	164.916	165.701
	166.362	172.38	185.435	192.198	194.284
3	0	346.240	352.503	352.792	355.018
	357.310	366.880	420.502	440.536	
2.5	0	574.337	584.916	586.165	589.786
	595.476	607.356	710.235	757.056	
2	0	1094.08	1115.94	1117.74	1129.12
	1148.10	1160.96	1379.94	1411.25	
1.5	0	2632.52	2690.39	2699.52	2756.73
	2910.76				
1.25	0	4703.20	4910.11	4933.83	5114.83
	5588.92				
1	0	7048.40	9844.57	11386.0	11635.8
	12269.8				
0.8	0	18828.9	25027.9		

The **Schonberg-Chandrasekhar Limit**

The maximum factional mass that can exist in the isothermal core.

$$\left(\frac{M_{ic}}{M}\right)_{\mathrm{SC}} \simeq 0.37 \left(\frac{\mu_{\mathrm{env}}}{\mu_{ic}}\right)^2,$$

(Read pages 453-456 for the derivation of S-C Limit.)

Example: Schonberg-Chandrasekhar Limit of a star

For a star with initial composition $X = 0.68$, $Y = 0.30$, and $Z = 0.02$, assuming complete ionization, what would be the μ_{env} ?

$$\frac{1}{\mu_i} = \sum_j \frac{1+z_j}{A_j} X_j \simeq 2X + \frac{3}{4}Y + \left\langle \frac{1+z}{A} \right\rangle_i Z.$$

$$\left\langle \frac{1+z}{A} \right\rangle_i \simeq \frac{1}{2}$$

$\mu_{env} \sim 0.63.$

At the core, all H is converted to He, $\mu_{ic} \sim 1.34$.

$$\left(\frac{M_{ic}}{M} \right)_{SC} \simeq 0.08.$$

[Try to calculate the mean molecular weight from scratch.]

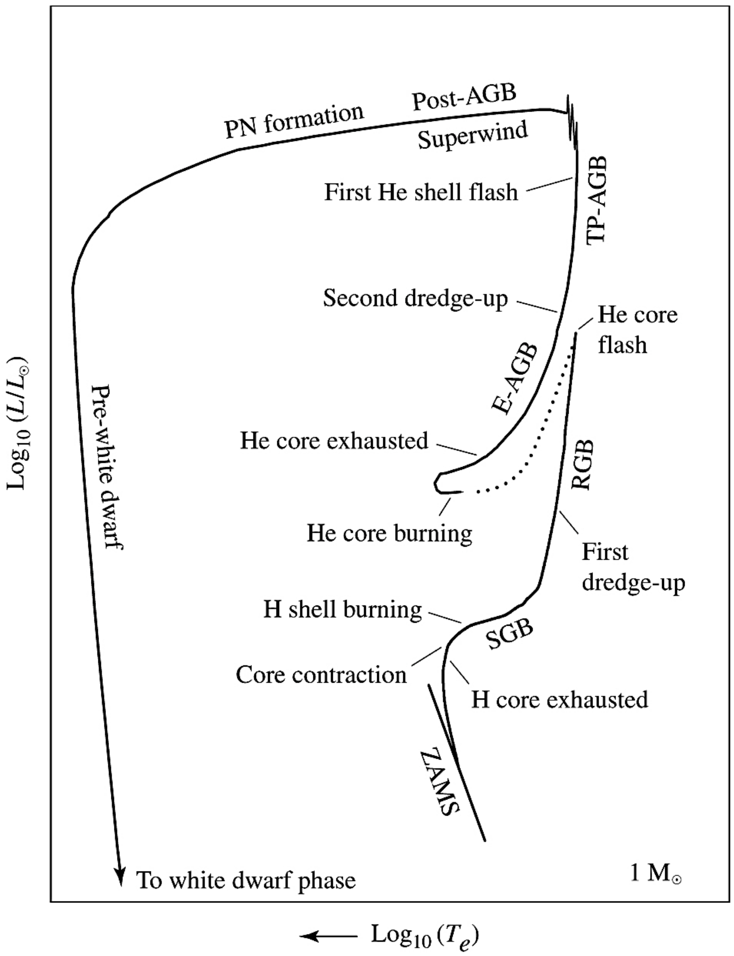
Degenerate Electron Gas

When the isothermal core exceeds the Schonberg-Chandrasekhar limit, the core will collapse unless there is an additional source of pressure.

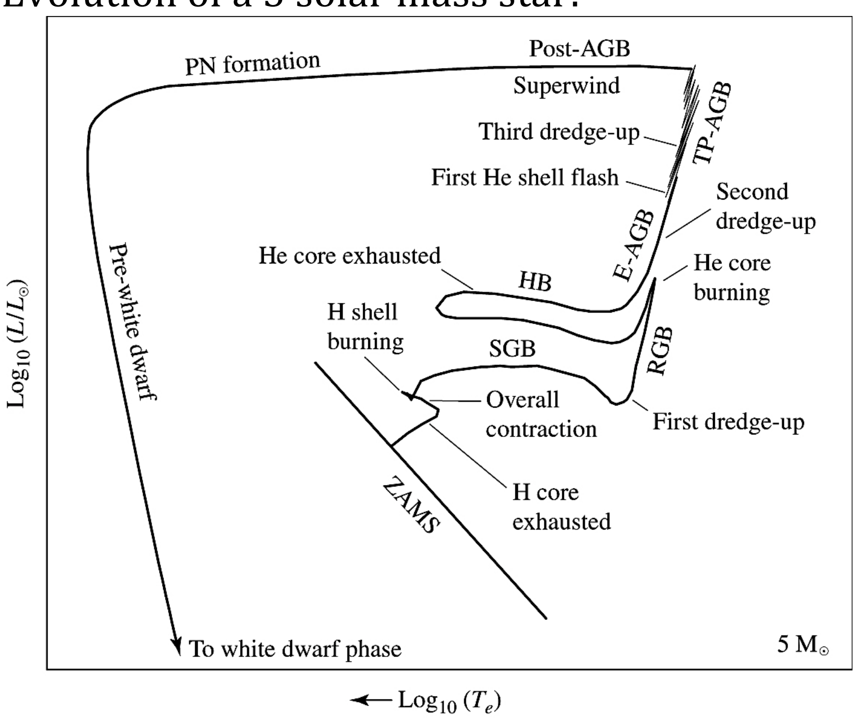
When the density is sufficiently high, the electrons are forced to occupy the lowest available energy levels. But electrons are Fermions that cannot occupy the same quantum state. Consequently, electrons have to occupy progressively higher energy states..

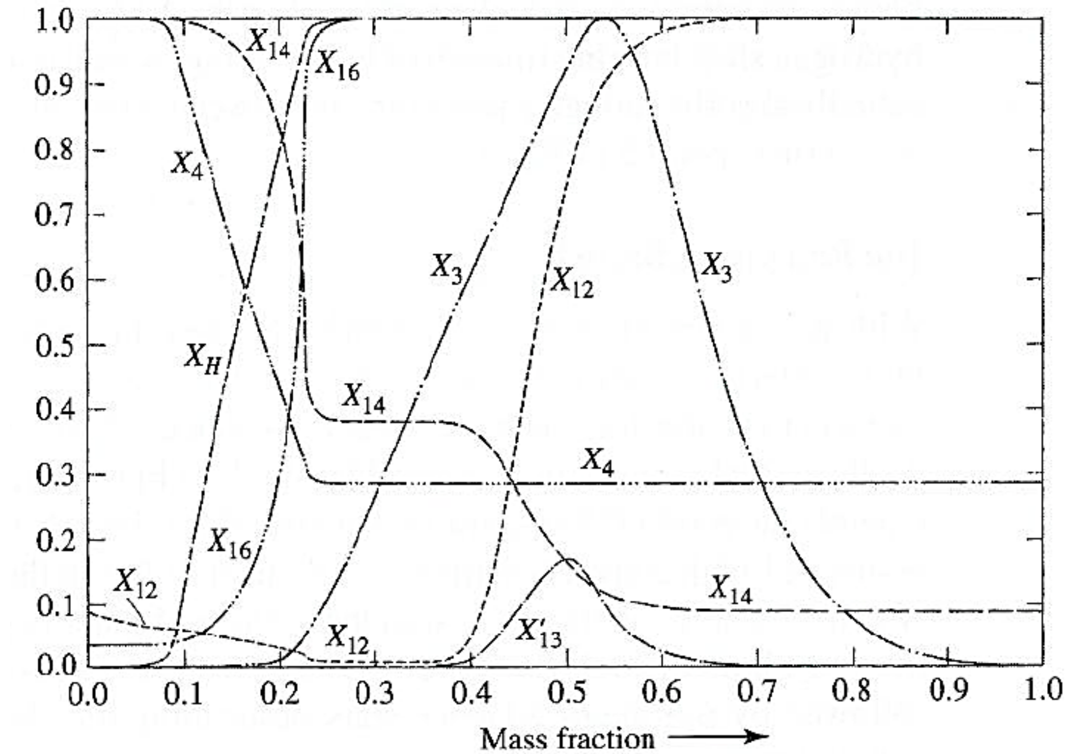
In the case of complete degeneracy, the pressure of the gas is provided by the nonthermal motion of the electrons, and the pressure becomes independent of temperature:

$$P_e = K\rho^{5/3}$$



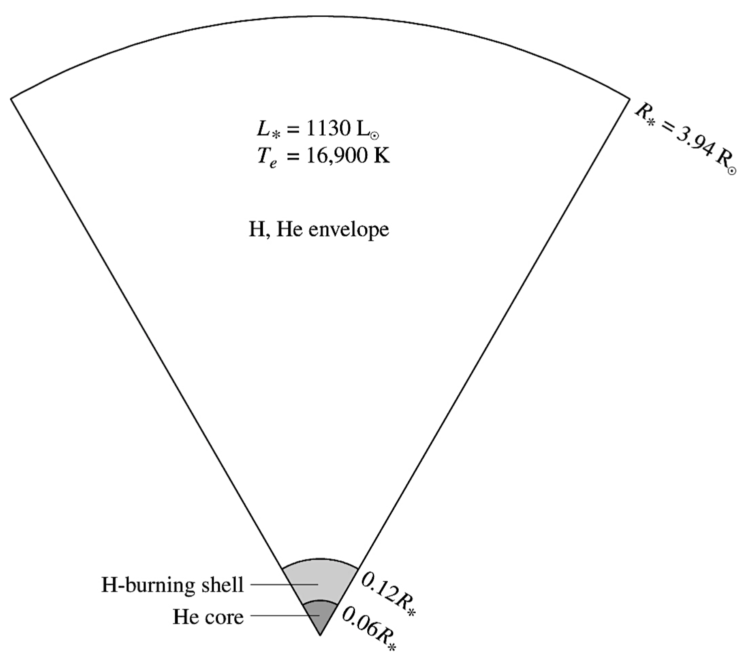
Evolution of a 5 solar mass star:

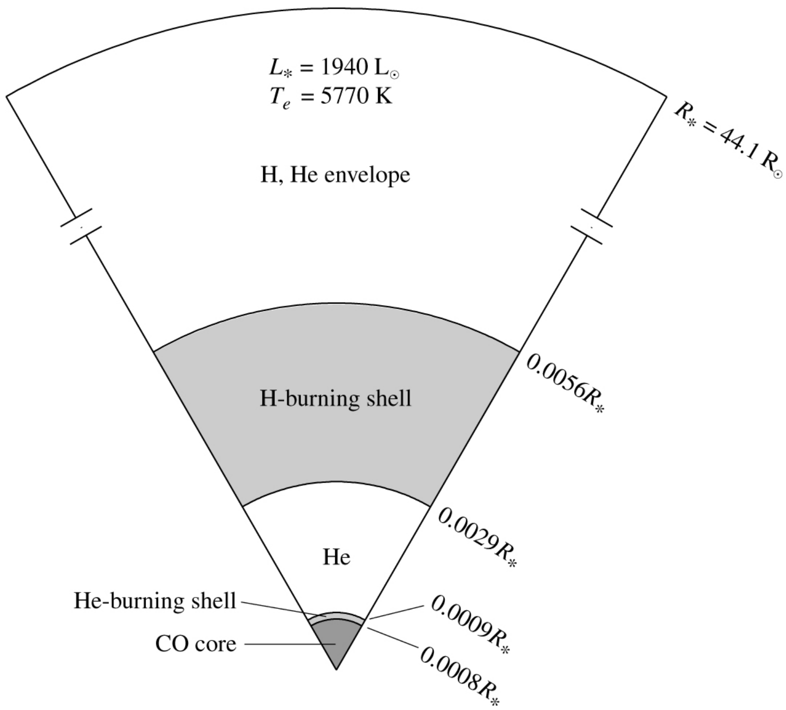




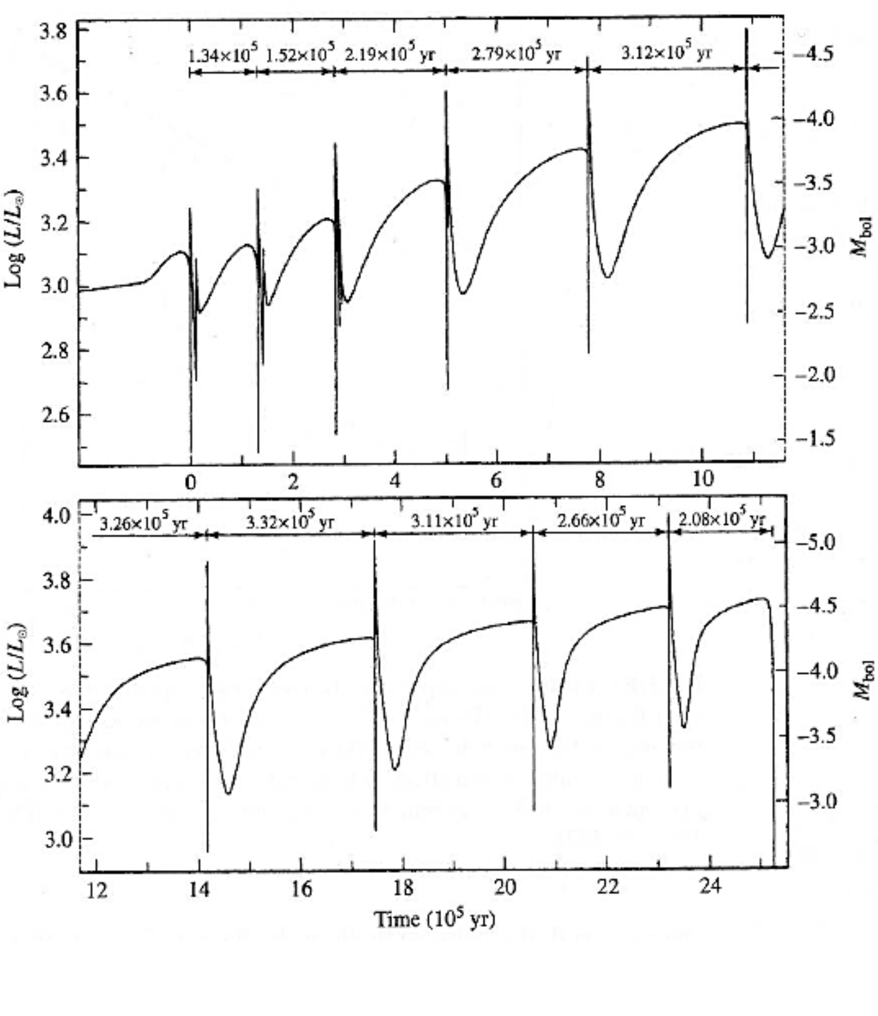
Above is the chemical composition of a 5 solar mass star after H is depleted in the core.

Structure/evolution of a 5 solar mass star:





$L_* = 1940\ L_\odot$
$T_e = 5770$ K
$R_* = 44.1\ R_\odot$
H, He envelope
H-burning shell
$0.0056R_*$
$0.0029R_*$
He
He-burning shell
CO core
$0.0009R_*$
$0.0008R_*$



1.34×10^5
1.52×10^5
2.19×10^5 yr
2.79×10^5 yr
3.12×10^5 yr
3.26×10^5 yr
3.32×10^5 yr
3.11×10^5 yr
2.66×10^5 yr
2.08×10^5 yr
Log ($L/L_\odot$)
M_{bol}
Time (10^5 yr)