Astronomy 404
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Chapter 12. The ISM and Star Formation

The interstellar medium (ISM) has many different phases (physical conditions – H_2, HI, HII, and HIM).

Star formation takes place mostly in cold molecular clouds. Once stars are formed, the stellar energy feedback dissociates, ionizes, heats, and dissipates the natal cloud.

Interstellar dust exists everywhere and causes extinction. The amount of dust correlates with the density of the interstellar gas, and molecular clouds have the highest densities, so molecular clouds contain a lot of dust and correlate well with dust clouds.

Extinction is a function of wavelength and is expressed as A_λ in the following formula:

$$m_\lambda = M_\lambda + 5\log_{10} d - 5 + A_\lambda$$

The extinction A_λ can be related to optical depth τ_λ .

$$I_\lambda / I_{\lambda,0} = e^{-\tau_\lambda}$$

Since $m_1 - m_2 = -2.5 \log_{10} (F_1/F_2)$, we have

$$m_\lambda - m_{\lambda,0} = -2.5\log_{10}\left(e^{-\tau_\lambda}\right) = 2.5\tau_\lambda \log_{10} e = 1.086\tau_\lambda$$

$$A_\lambda = 1.086\tau_\lambda$$

The change in magnitude due to extinction is approximately equal to the optical depth along the line of sight.

For a number density of the dust grains n_d and a scattering cross section σ_λ, the optical depth is

$$\tau_\lambda = \int_0^s n_d(s')\,\sigma_\lambda\,ds'$$

If σ_λ is constant along the line of sight, then

$$\tau_\lambda = \sigma_\lambda \int_0^s n_d(s')\,ds' = \sigma_\lambda N_d$$

where N_d is the dust grain column density.

What is σ_λ? We need **Mie Theory** by Gustav Mie in 1908.

The cross section of a spherical dust grain with radius a is $\sigma_g = \pi a^2$. The dimensionless **extinction coefficient** Q_λ is

$$Q_\lambda \equiv \frac{\sigma_\lambda}{\sigma_g}$$

Q_λ depends on the composition of the dust grains.

Mie showed that when the wavelength of the light is on the order of the size of the dust grains, then $Q_\lambda \sim a/\lambda$, and

$$\sigma_\lambda \propto \frac{a^3}{\lambda} \qquad (\lambda \gtrsim a)$$

When λ is much larger than a, Q_λ goes to zero.

If λ is much smaller than a, then Q_λ approaches a constant:

$$\sigma_\lambda \propto a^2 \qquad (\lambda \ll a)$$

Example:

A star located at 0.8 kpc from the Earth has an extinction of A_V = 1.1 mag at 550 nm. If Q_{550} = 1.5 and the dust grains are spherical with a = 0.2 μm, what is the average density of material along the line of sight?

$$\tau_\lambda = A_\lambda/1.086 = 1.1/1.086 \sim 1.$$

$$\sigma_{550} = \pi a^2 Q_{550} \simeq 2 \times 10^{-13}\ \mathrm{m}^2$$

$$N_d = \frac{\tau_{550}}{\sigma_{550}} \simeq 5 \times 10^{12}\ \mathrm{m}^{-2}$$

$$N_d = \int_0^s n(s')\, ds' = \overline{n} \times 0.8\ \mathrm{kpc}$$

Therefore,

$$\overline{n} = \frac{N_d}{0.8\ \mathrm{kpc}} = 2 \times 10^{-7}\ \mathrm{m}^{-3}$$

Molecular clouds are often classified according to their interstellar extinction:

$1 < A_V < 5$	**diffuse (translucent) molecular cloud** $T \sim$ 15 to 50 K $n \sim 5\times10^8$ to 5×10^9 m^{-3} $M \sim$ 3 to 100 $M_\odot$
	giant molecular clouds (GMCs) $T \sim$ 15 K $n \sim (1\text{-}3) \times10^8$ m^{-3} $M \sim 10^5$ to 10^6 $M_\odot$ size $\sim$ 50 pc
$A_V \sim 5$	**dark cloud complex** $T \sim$ 10 K $n \sim 5 \times10^8$ m^{-3} $M \sim 10^4$ $M_\odot$ size $\sim$ 10 pc
$A_V \sim 10$	**clumps** $T \sim$ 10 K $n \sim 10^9$ m^{-3} size $\sim$ 1-2 pc
$A_V > 10$	**dense cores** $T \sim$ 10 K $n \sim 10^{10}$ m^{-3} $M \sim$ 10 $M_\odot$ size $\sim$ 0.1 pc

$A_V \sim 10$ **Bok globules**

$T \sim 10$ K
$n > 10^{10}$ m^{-3}
$M \sim 1\text{-}10^3\ M_\odot$
size < 1 pc

The dense molecular regions are where star formation is about to start or has already started.

The Formation of Protostars

To determine the conditions for a cloud to collapse to form stars, we star with the virial theorem:

$$2K + U = 0$$

where K is the total internal kinetic energy and U the gravitational potential energy.

For a spherical cloud of uniform density

$$U \sim -\frac{3}{5}\frac{GM_c^2}{R_c}$$

$$K = \frac{3}{2}NkT$$

$$N = \frac{M_c}{\mu m_H}$$

For a cloud to collapse, $2K + U < 0$, or $2K < |U|$

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\frac{GM_c^2}{R_c}$$

$$R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}$$

Jeans criterion: $M_c > M_J$, **Jeans mass** M_J

$$M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$R_c > R_J$, **Jeans length** R_J

$$R_J \simeq \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2}$$

The Jeans criterion is derived with zero pressure on the surface of the cloud.

If the external pressure P_0 is considered, the critical mass for the cloud collapse is given by the **Bonnor-Ebert mass,**

$$M_{\mathrm{BE}} = \frac{c_{\mathrm{BE}} v_T^4}{P_0^{1/2} G^{3/2}},$$

where $v_T \equiv \sqrt{kT/\mu m_H}$ and $c_{\mathrm{BE}} \simeq 1.18$.

The Jeans mass can be expressed in the same form, but $c_J \sim 5.46$.

Bonnor-Ebert mass is smaller than Jeans mass because of the external pressure, which makes cloud collapse easier to happen.

Once the collapse starts, one can calculate the free-fall time scale by assuming a homologous collapse.

$$\frac{d^2r}{dt^2} = -G\frac{M_r}{r^2}$$

$$\frac{dr}{dt}\frac{d^2r}{dt^2} = -\left(\frac{4\pi}{3}G\rho_0 r_0^3\right)\frac{1}{r^2}\frac{dr}{dt}$$

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \left(\frac{4\pi}{3}G\rho_0\, r_0^3\right)\frac{1}{r} + C_1$$

$$C_1 = -\frac{4\pi}{3}G\rho_0\, r_0^2$$

$$\frac{dr}{dt} = -\left[\frac{8\pi}{3}G\rho_0\, r_0^2\left(\frac{r_0}{r} - 1\right)\right]^{1/2}$$

$$\theta \equiv \frac{r}{r_0} \qquad\qquad \chi \equiv \left(\frac{8\pi}{3}G\rho_0\right)^{1/2}$$

$$\frac{d\theta}{dt} = -\chi\left(\frac{1}{\theta} - 1\right)^{1/2}$$

$$\theta \equiv \cos^2\xi$$

$$\cos^2\xi\,\frac{d\xi}{dt} = \frac{\chi}{2}$$

$$\frac{\xi}{2} + \frac{1}{4}\sin 2\xi = \frac{\chi}{2}t + C_2$$

At $t = 0$, $r = r_0$, $\theta = 1$, $\xi = 0$, therefore, $C_2 = 0$.

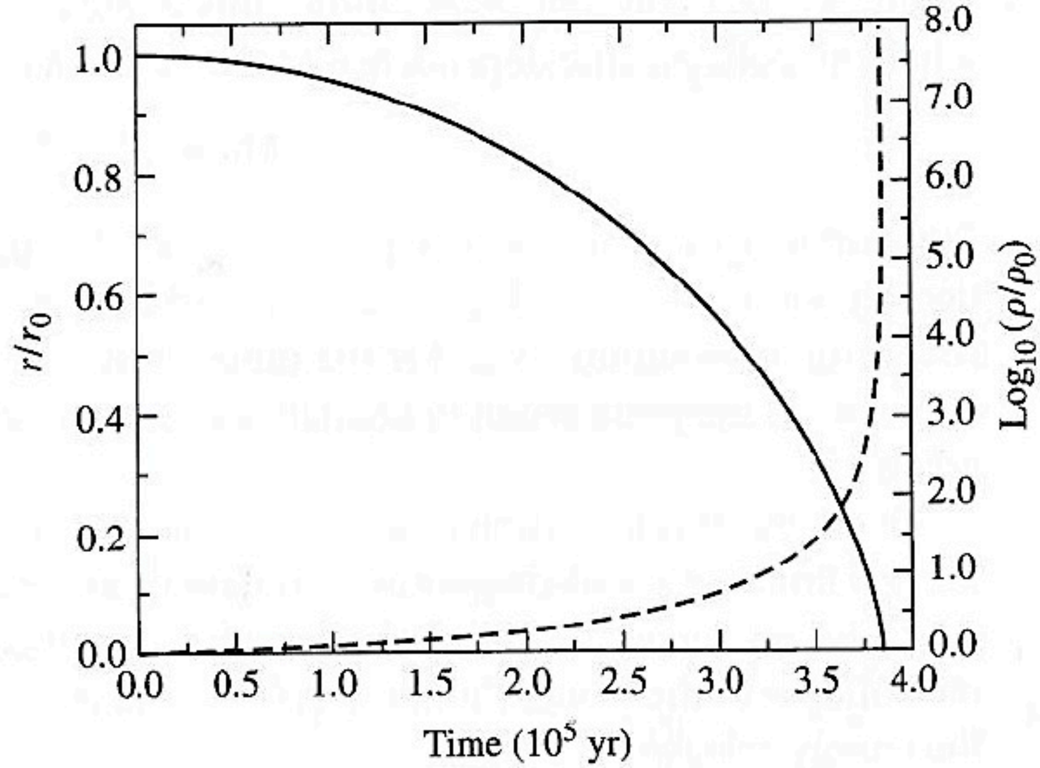
$$\xi + \frac{1}{2}\sin 2\xi = \chi t$$

When $r = 0$, $t = t_{ff}$, ($\theta = 0$, $\xi = \pi/2$), therefore,

$$t_{ff} = \frac{\pi}{2\chi}$$

$$\boxed{t_{ff} = \left(\frac{3\pi}{32}\frac{1}{G\rho_0}\right)^{1/2}.}$$

This is for homologous collapse, meaning that everywhere it takes the same amount of time to collapse and the density increases at the same rate everywhere. However, this does not happen in reality – the free-fall time is shorter near the center than for material further out. **Inside-out collapse**.



The above figure shows an example of a homologous collapse (Example 12.2.2 of Carroll & Ostlie).

Fragmentation of collapsing clouds

Jeans mass:

$$M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

As clouds collapse, density gets higher, so M_J decreases and fragment. When the cloud becomes optically thick, and T increases, M_J goes up again and stops further fragmentation and form stars of some mass.