Astronomy 404
October 14, 2013

Equations for Stellar Structure

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}$$

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2} \qquad \text{(radiation)}$$

$$= -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} \qquad \text{(adiabatic convection)}$$

Convection occurs when $\frac{d\ln P}{d\ln T} < \frac{\gamma}{\gamma - 1}$, then the convection temperature gradient is purely adiabatic.

The Vogt-Russell Theorem

The mass and composition structure of a star uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution.

The Main Sequence

The vast majority of all stars consist of primarily hydrogen ($X \sim 0.7$), whereas the mass fraction of metals varies from nearly zero to 3% ($0 < Z < 0.03$).

The first nuclear reaction is hydrogen burning (PP chain or CNO cycle) because its Coulomb barrier is lower than those of more massive nuclei.

Most stars have similar initial compositions. Stars with higher masses would have higher central pressure and temperatures. PP Chain dominates in low-mass stars. CNO cycle dominates in stars slightly more massive than the Sun (~1.2 $M_\odot$).

Hydrogen burning is a relatively slow process and star has lots of hydrogen, a star spends most of its life burning hydrogen.

The smallest mass for a star whose central temperature is still hot enough for nuclear reaction is ~0.072 $M_\odot$ for solar abundance, and ~0.09 $M_\odot$ for $Z \sim 0$.

The highest mass a star is limited by thermal oscillation to 90 $M_\odot$ and also limited by radiation pressure on the surface – the *Eddington luminosity limit.*

Radiation pressure gradient drives the radiative flux

$$\frac{dP_{\text{rad}}}{dr} = -\frac{\overline{\kappa}\rho}{c} F_{\text{rad}} \quad \longrightarrow \quad \frac{dP}{dr} \simeq -\frac{\overline{\kappa}\rho}{c}\frac{L}{4\pi r^2}$$

$$\frac{dP}{dr} = -G\frac{M\rho}{r^2}$$

When the radiation pressure force balances the gravitational force, the star is in hydrostatic equilibrium.

$$-\frac{\overline{\kappa}\rho}{c}\frac{L}{4\pi r^2} = -G\frac{M\rho}{r^2}$$

The Eddington Limit:

$$\boxed{L_{\mathrm{Ed}} = \frac{4\pi G c}{\overline{\kappa}} M.}$$

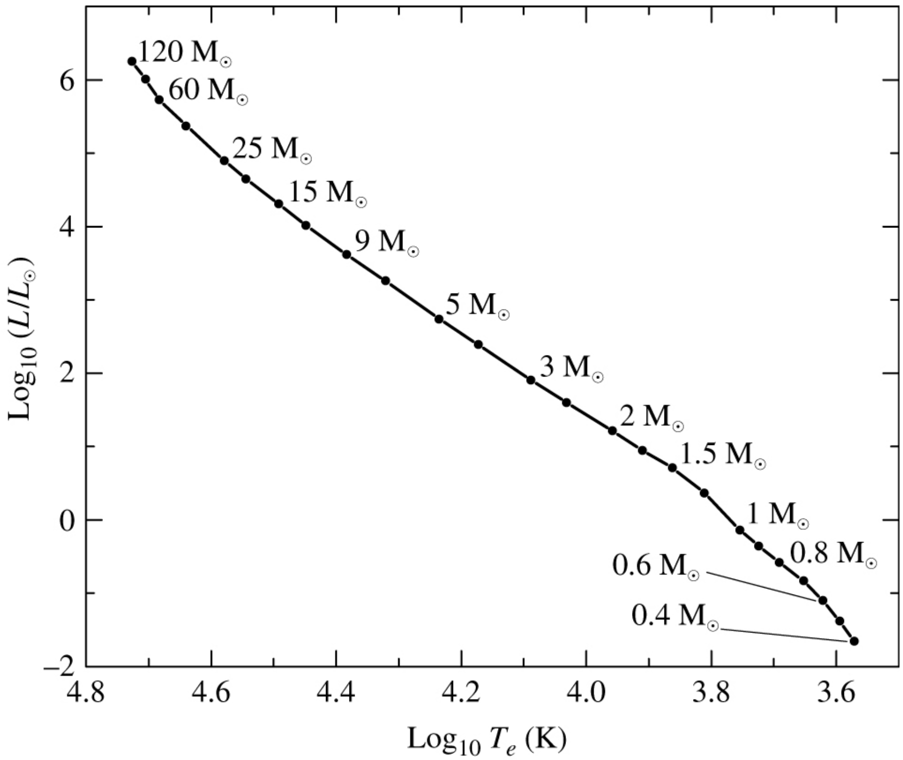
Massive stars with effective temperatures ~ 50,000 K,
H and He are totally ionized, so the opacity is dominated by electron scattering,

$$\overline{\kappa}_{\mathrm{es}} = 0.02(1+X)\ \mathrm{m^2\ kg^{-1}} = 0.034\ \mathrm{m^2\ kg^{-1}}$$

$$L_{\mathrm{Ed}} \simeq 1.5 \times 10^{31} \frac{M}{M_\odot}\ \mathrm{W}$$

$$\frac{L_{\mathrm{Ed}}}{L_\odot} \simeq 3.8 \times 10^4 \frac{M}{M_\odot}$$

For a 90 $M_\odot$ star, $L_{\mathrm{Ed}} \simeq 3.5 \times 10^6\ L_\odot$, roughly 3 times the expected main sequence luminosity.



M	0.072 $M_\odot$	90 $M_\odot$
T	1700 K	53,000 K
L	5×10^{-4} $L_\odot$	1×10^{6} $L_\odot$

PP Chain dominates for stars with $M < 1.2\ M_\odot$

CNO Cycle dominates for stars with $M > 1.2\ M_\odot$ and result in convection in the stellar core.

Stars with $M < 1.3\ M_\odot$ have effective temperatures lower than 8000 K. In their interiors where $T \sim 10{,}000$ K, photoionization of H raises the opacity and causes convection. Low-mass stars have convective envelopes.

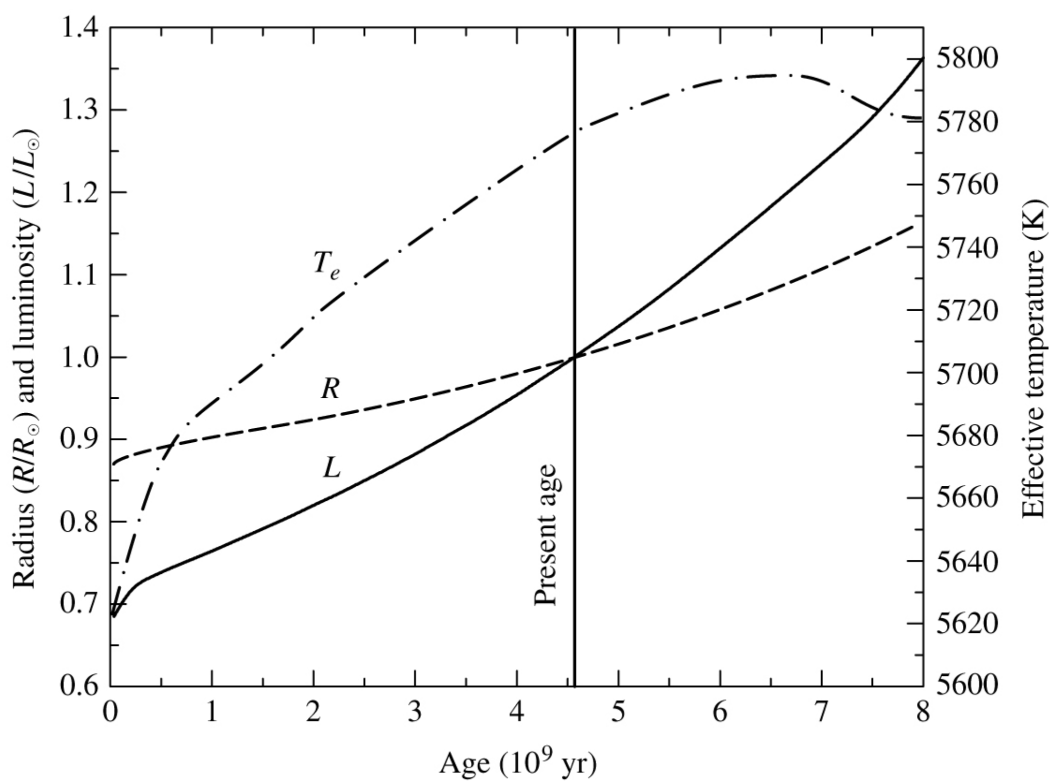
Chapter 11. The Sun

The solar interior

The Sun is a G2 V star with a surface composition of $X = 0.74$, $Y = 0.24$, and $Z = 0.02$. This composition is not original; it has been modified by elemental diffusion (gravitational settling).

The ages of moon rocks and meteorites determined by radioactive dating indicate that the Sun is $\sim 4.57\times10^9$ yr old.

Evolution model of a 1 $M_\odot$ star that produces the observed properties of the Sun at 4.6×10^9 yr show the following:

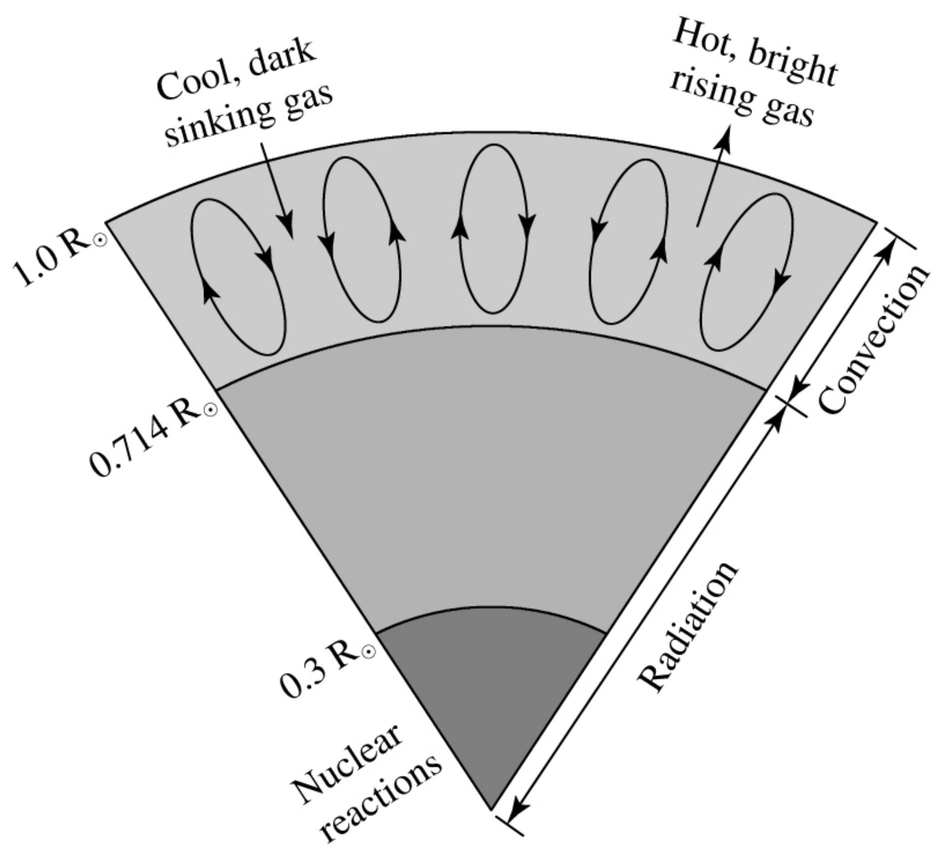


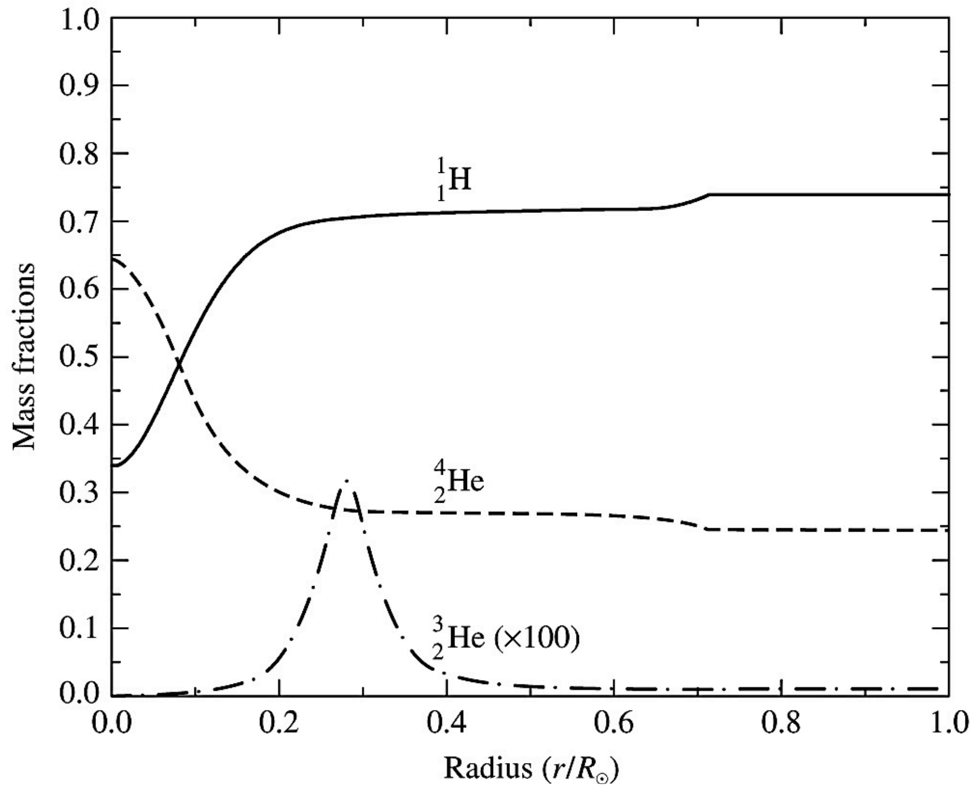
The Sun's luminosity increased 48%, radius 15%, and temperature 2.8% (5620 to 5777 K).

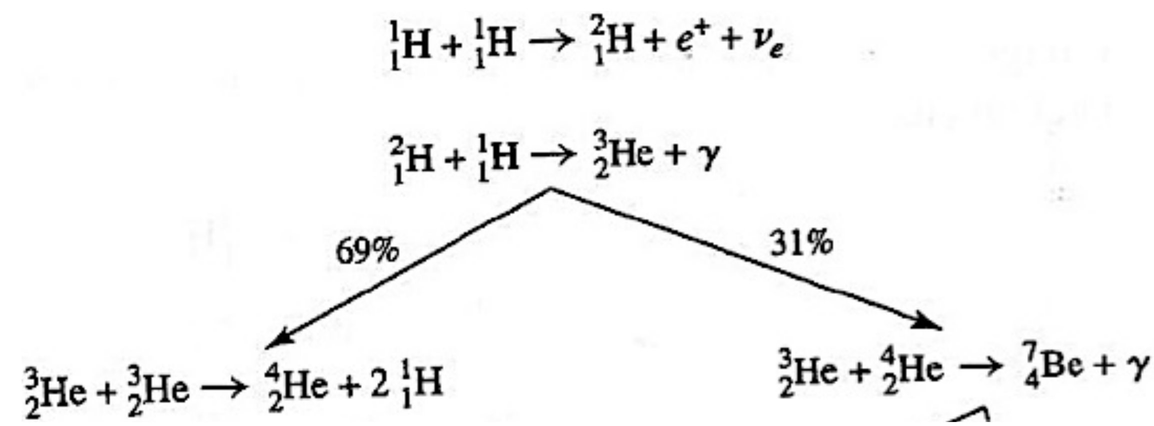
The present-day Sun has these central conditions:

Temperature	1.570×10^7 K
Pressure	2.342×10^{16} N m^{-2}
Density	1.527×10^5 kg m^{-3}
X	0.3397
Y	0.6405

The initial *X* was 0.71, and the initial *Y* was 0.27.
The solar surface now has X = 0.74, *Y* = 0.24; the changes are caused by diffusive settling of heavier elements.

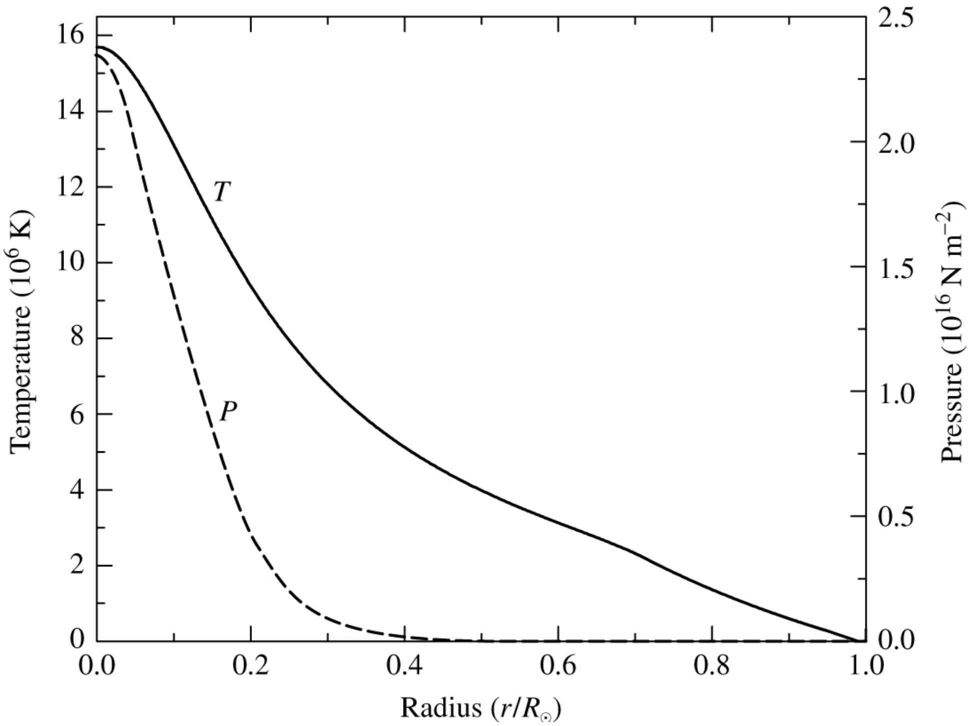


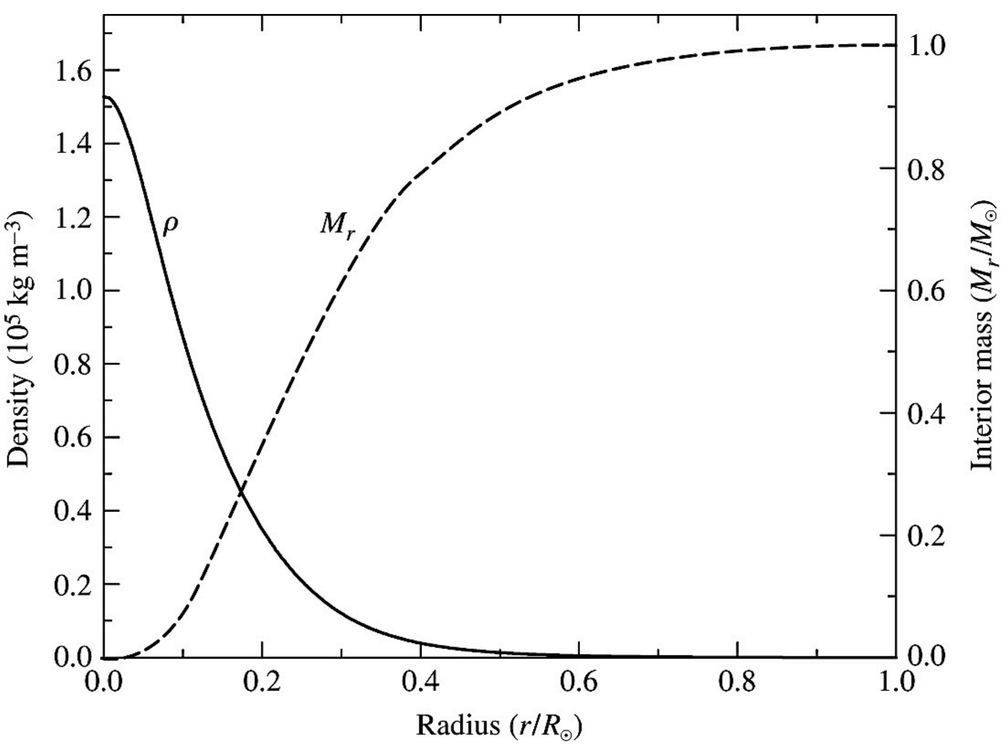




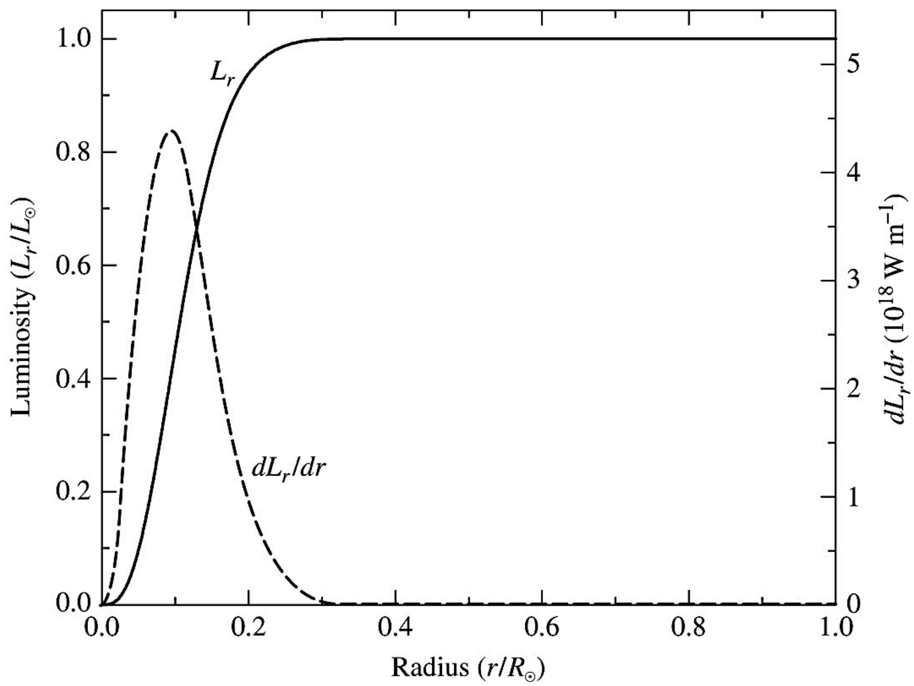
^3_2He is produced and then destroyed again. At the top of the H-burning region where the temperature is lower, ^3_2He is relatively more abundant because it is produced more easily than it is destroyed.

The bump of ^1H and ^4He at 0.7 $R_\odot$ is caused by convection and elemental diffusion.



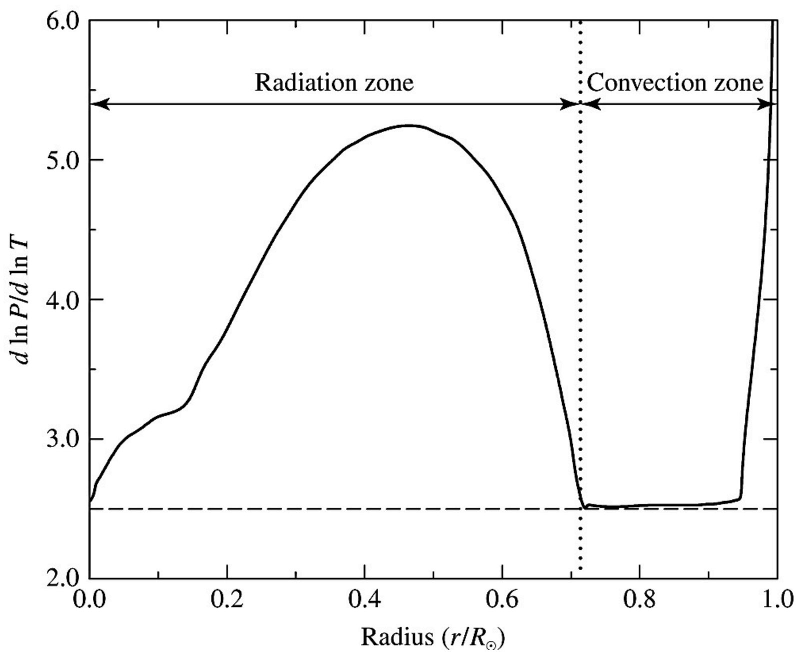


90% of the mass is located within ~ 0.5 $R_\odot$!



The largest energy production occurs at $R = 0.1R_\odot$, because of a combination of r, T, and X :

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

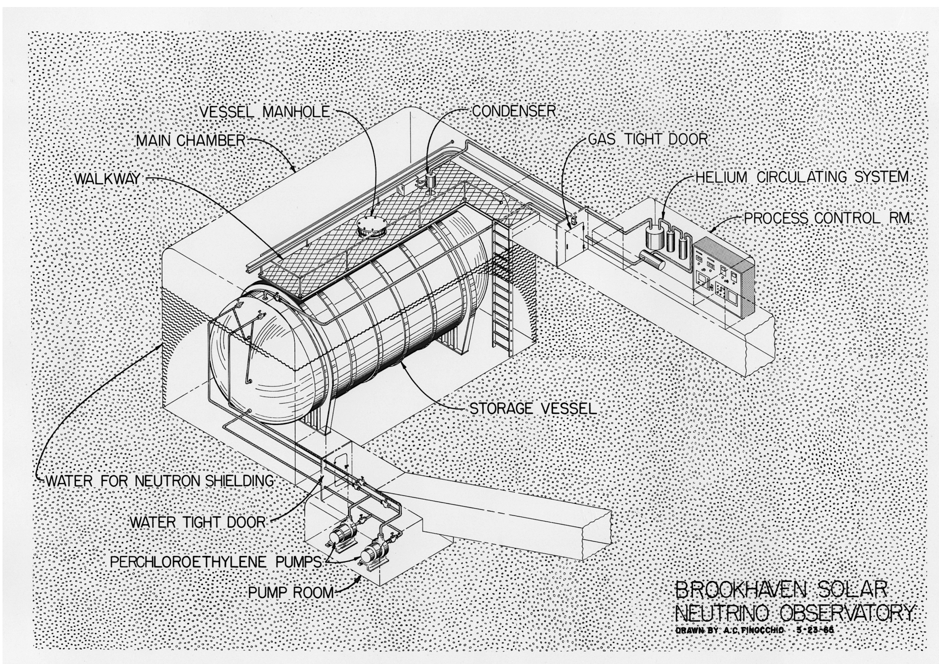


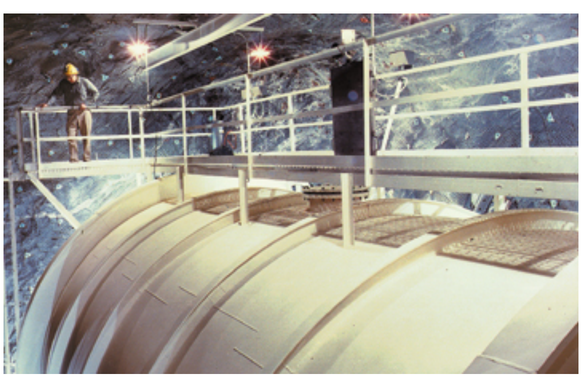
For an ideal monatomic gas, $\gamma = 5/3$, and convection occurs when $d \ln P \,/\, d \ln T < \gamma/(\gamma-1) = 2.5$.

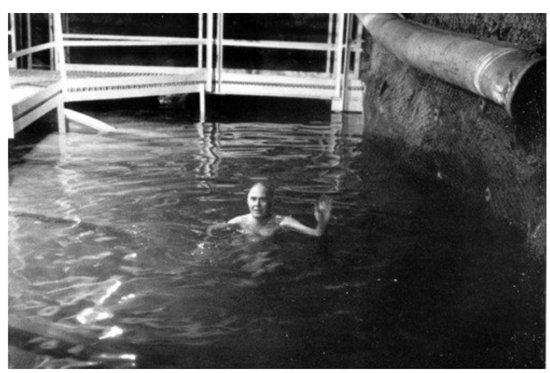
There were two outstanding problems with the solar model: *lithium* and *neutrino.* The lithium problem will be discussed in Chapter 13. Now we deal with the solar neutrino problem.

The Solar Neutrino Problem

Raymond Davis was told by his supervisor at Brookhaven National Lab to find something interesting to work on, and he decided to develop methods to detect solar neutrinos… One mile below ground in the Homestake Gold Mine in South Dakota.







Raymond Davis, Jr. would be 99 yr old today!

615,000 kg of cleaning fluid, C_2Cl_4, in a volume of 100,000 gallons.

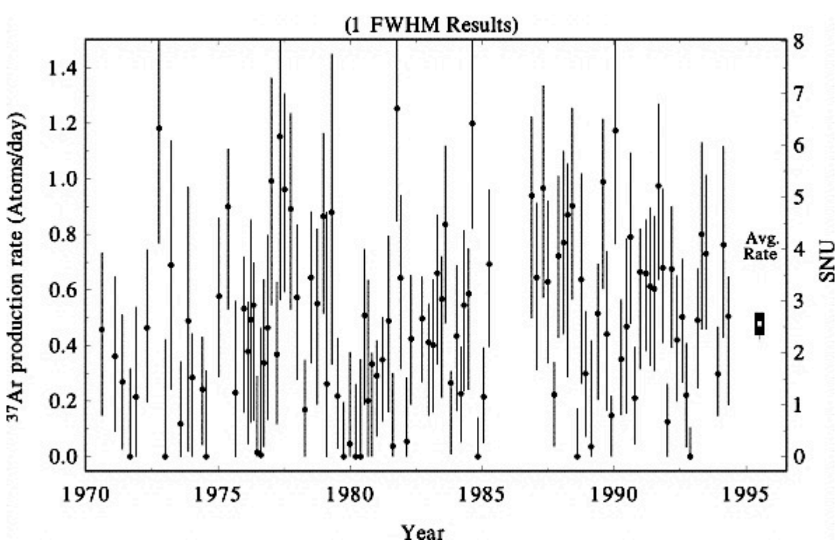
$$^{37}_{17}\mathrm{Cl} + \nu_e \rightleftharpoons {}^{37}_{18}\mathrm{Ar} + e^-$$

The radioactive isotope of Ar has a half-life of 35 days.

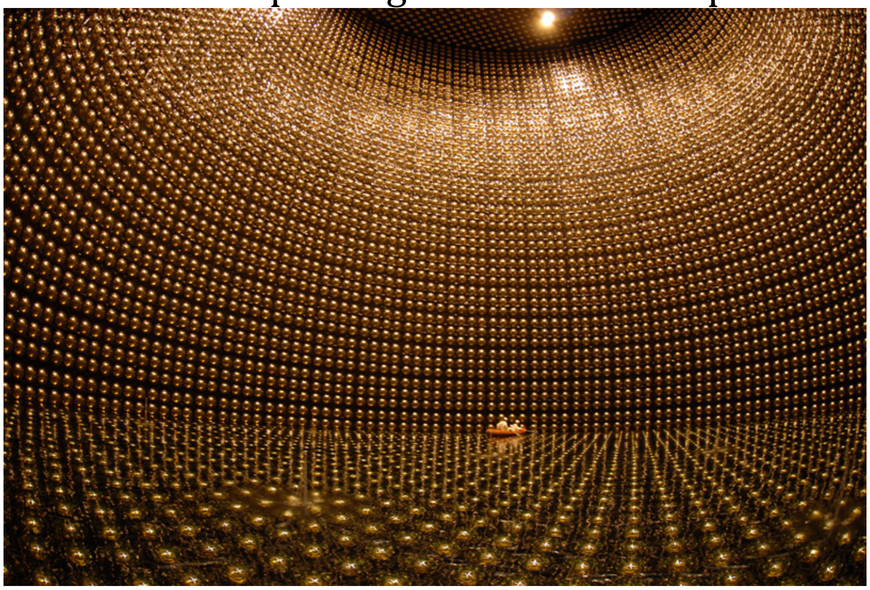
PP I produces neutrinos with 0.26 MeV energy
PP II produces neutrinos with 0.80 MeV energy
PP III produces neutrinos with 7.2 MeV energy

There should be many more neutrinos from PP I, but 77% of the neutrinos detected in the Davis experiment were produced by PP III.

Once every few months, they purge the tank and determine the number of argon atoms produced. The capture rate was measured in terms of the **solar neutrino unit,** or **SNU**
(1 SNU = 10^{-36} reactions per target atom per second).
With ~ 2.2×10^{30} atoms of $^{37}_{17}\mathrm{Cl}$ in the tank, if 1 Ar atom is produced each day, the rate would be 5.35 SNU.
However, only 1 Ar atom was produced every two days in that 100,000 gallon tank, corresponding to 2.56± 0.16 SNU.



Super-Kamiokande Neutrino Observatory detects the Cerenkov light produced when neutrinos scatter electrons, causing the electrons to move at speeds greater than the speed of light in water.



The Soviet-American Gallium Experiment (SAGE) in Caucasus and the GALLEX experiment in Italy make their detections via

$$\nu_e + {}^{71}_{31}\mathrm{Ga} \rightarrow {}^{71}_{32}\mathrm{Ge} + e^-$$

They also confirm the deficiency of neutrinos reported by Davis.

The solution: **Mikheyev-Smirnov-Wolfenstein (MSF) effect**

There are three flavors of neutrinos: electron neutrino, muon neutrino, and tau neutrino. The MSF effect suggests that neutrinos oscillate among these three flavors as they pass through the solar interior and interact with electrons.

PP chain produces only electron neutrinos, and the Homestake, Super-Kamiokande, SAGE, and GALLEX all detect electron neutrinos; however, because of the neutrino oscillation, only a fraction of solar neutrinos emerge as electron neutrinos.

Neutrinos can change flavor only if they have masses. The upper limit of the electron neutrino is ~2.2 eV.

Confirmation of the neutrino oscillation came in 1998 when Super-Kamiokande was used to detect atmospheric neutrinos produced by high-energy cosmic rays.

Cerenkov light shows direction, so Super-Kamiokande can measure muon neutrinos traveling downward and the muon neutrinos traveling upward (through the Earth). The number of muon neutrinos through the Earth is smaller, in excellent agreement with the expectation of neutrino oscillation among the three flavors.

Raymond Davis, Jr. and Masatoshi Koshiba received 2002 Nobel Prize in Physics.   Davis passed away in 2006. Today, Oct 14, is his birthday.