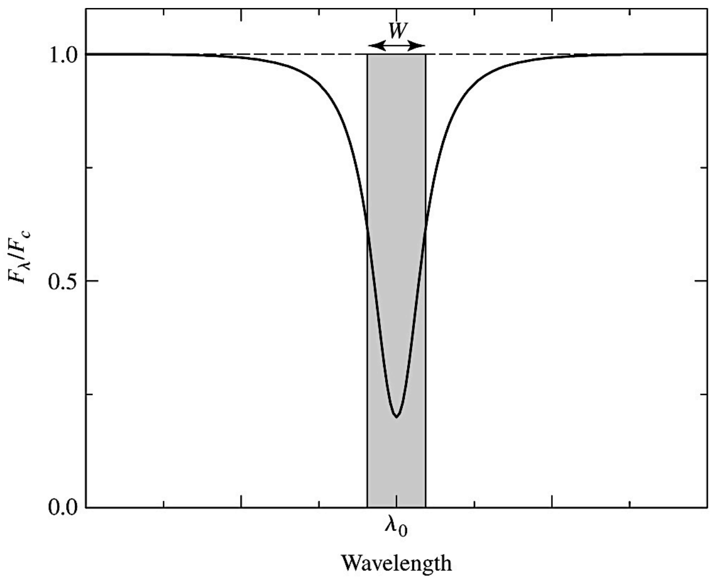
Astronomy 404
September 27, 2013

Chapter 9. Stellar Atmospheres

The Profiles of Spectral Lines



Equivalent width of a line $$W = \int \frac{F_c - F_\lambda}{F_c}\, d\lambda.$$

Core and wings of a line
Optically thick or thin

Processes that Broaden Spectral Lines

1. Natural Broadening (Heisenberg's uncertainty principle)

$$\Delta E \approx \frac{\hbar}{\Delta t} \qquad \Delta\lambda \approx \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$$

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c}\frac{1}{\Delta t_0}$$

Δt_0 : average waiting time for a transition
$(\Delta\lambda)_{1/2}$ is typically ~ 2.4×10^{-5} nm

2. Doppler broadening

$$v_{mp} = \sqrt{2kT/m}$$

$$\Delta\lambda/\lambda = \pm |v_r|/c$$

$$\Delta\lambda \approx \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}$$

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT \ln 2}{m}}$$

For hydrogen in the Sun's atmosphere (T = 5777 K), the Doppler broadening of the Hα line is $\Delta\lambda \sim 0.0427$ nm, roughly 1000 times greater than for natural broadening.

Doppler broadening contributes to the core of a line. It decreases exponentially due to the Maxwell-Boltzmann velocity distribution.

In stellar atmosphere, there could be a large-scale turbulent motion of large masses of gas, the line width becomes

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\left(\frac{2kT}{m} + v_{turb}^2\right) \ln 2}$$

The turbulent component is particularly important in the atmospheres of giants and supergiants.

Other Doppler broadenings involve stellar rotation, pulsation, and mass loss (P Cygni profile).

3. Pressure (and collisional) broadening

The orbitals of an atom can be perturbed in a collision with a neutral atom or by a close encounter of an ion. The results of individual collisions are called *collisional broadening*, and the statistical effects of a large number of closely passing ions are called *pressure broadening*.

The precise calculation of the shape of pressure-broadened line is very complicated, involving ions and atoms of different elements and free electrons in collisions and close encounters.
The general shape of a pressure-broadened line is similar to that of the natural broadening, called a *damping profile* (also known as *Lorentz profile*), so named because the shape is characteristic of a spectrum of radiated by an electric charge undergoing damped simple harmonic motion.

Recall the line width from natural broadening:

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c} \frac{1}{\Delta t_0}$$

$$\Delta t_0 \approx \frac{\ell}{v} = \frac{1}{n\sigma\sqrt{2kT/m}}$$

the mean free path is $1/n\sigma$, σ is the collision cross section, n is the number density of the atom, and m is the mass of the atom.

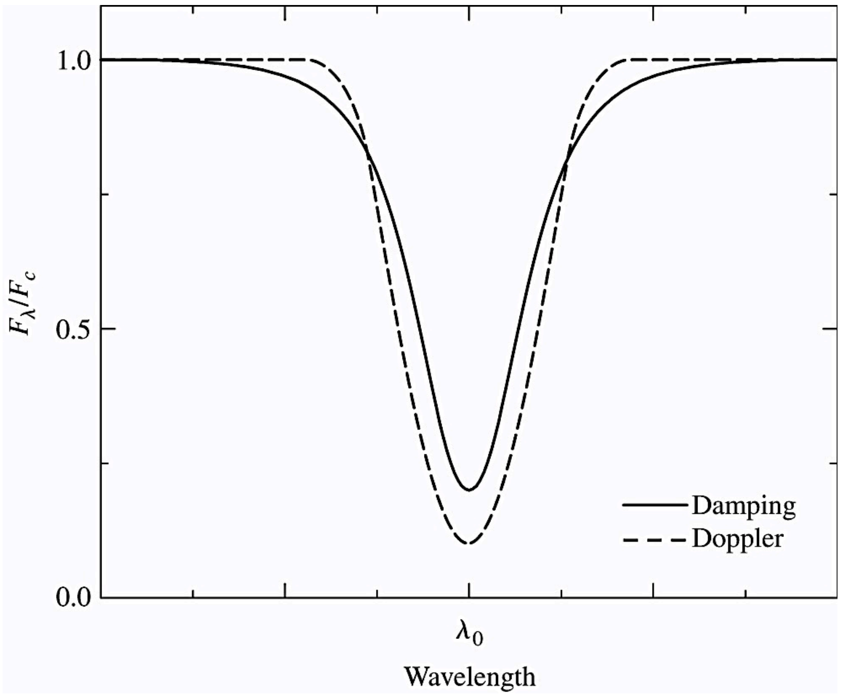
$$\Delta\lambda = \frac{\lambda^2}{c} \frac{1}{\pi \Delta t_0} \approx \frac{\lambda^2}{c} \frac{n\sigma}{\pi} \sqrt{\frac{2kT}{m}}$$

The pressure broadening is proportional to density n; therefore, giants and supergiants with larger radii and lower densities in the atmospheres have narrower lines.
(physical reason for the Morgan-Keenan luminosity classes)

In the Sun's atmosphere, n of hydrogen atom is 1.5×10^{23} m^{-3}.
The pressure broadening of the Hα line is $\Delta\lambda \sim 2.36\times10^{-5}$ nm, comparable to natural broadening. For comparison, Doppler broadening has $\Delta\lambda \sim 0.0427$ nm.

The Voigt Profile

The total line profile, called a Voigt profile, is due to the contributions of both the Doppler and damping profiles.



The line profile has a *Doppler core* and *damping wings*.

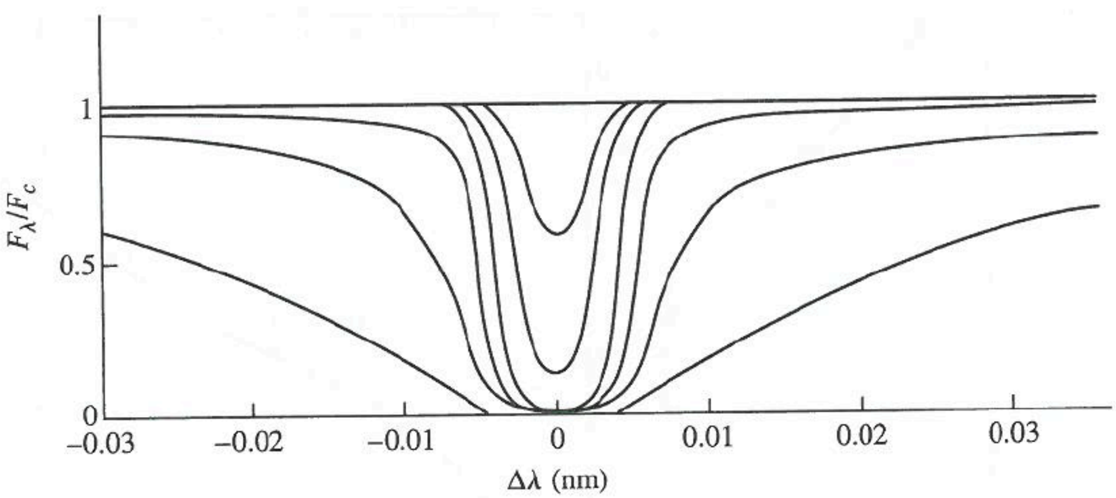
Luminosity class VI – subdwarf stars
They are deficient in metals ➔ fewer free electrons ➔ reducing the number of H^- ➔ lower continuum opacity ➔ seeing deeper into the atmosphere ($\tau = 2/3$) ➔ higher temperature

Therefore, subdwarfs are not less luminous than the main sequence dwarfs. They are hotter than the dwarfs of the same luminosity. Lower abundance ➔ lower opacity ➔ bluer color

Schuster-Schwarzschild model of line formation:
assuming blackbody radiation from the photosphere and absorption line formation in the atmosphere.

Column density of an absorbing atom - N_a (units: atoms m^{-2})
The relative probabilities of an electron making a transition from the same initial orbital are given by the *f-values* or *oscillator strengths* for the orbital.

The oscillator strength is the effective number of electrons per atom participating in the transition, thus $f N_a$ is the column density of atoms actually absorb to form the line.

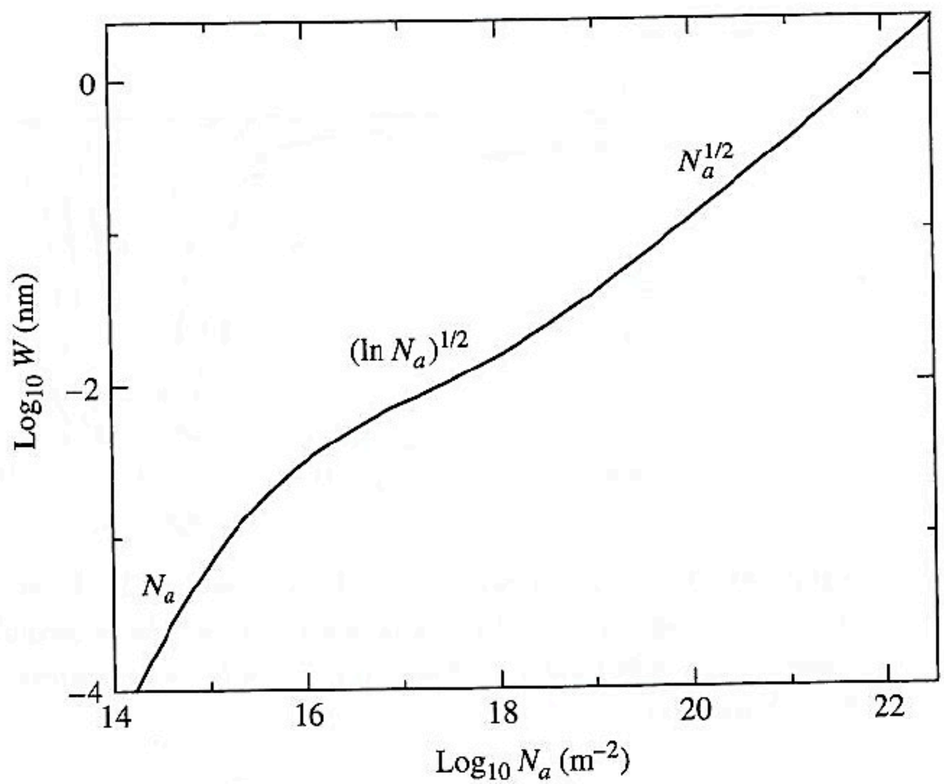


Voigt profiles of the Ca II K line (λ = 393.3 nm)
$N_a = 3.4\times10^{15}$ ions m^{-2}, increases by a factor of 5 successively.

Modern-day astronomers use stellar atmosphere models to calculate spectra, adjusting the abundances, surface gravity, and stellar effective temperature to match the observed spectra. You get all these parameters in the best-fit model.

However, in the old days before personal computers were available…

The Curve of Growth



First part – linear growth, optically thin

$$W \propto N_a$$

Second part - slower growth, optically thick

$$W \propto \sqrt{\ln N_a}$$

Third part - pressure-broadening comes in…

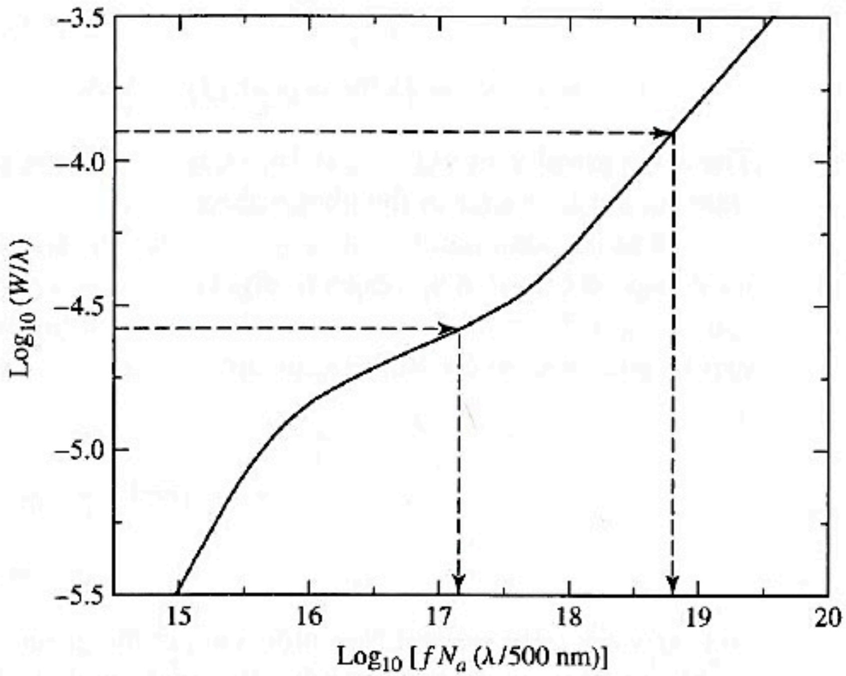
$$W \propto \sqrt{N_a}$$

It is advantageous to use several lines formed by transitions from the same initial orbital. Thus, the curve of growth is also plotted in $\log_{10}(W/\lambda)$ vs $\log_{10}\,[f N_a\,(\lambda/500\text{ nm})]$, as shown in the next page.

Example: Find the column density of Na in the Sun's atmosphere.

NaI lines at 330.238 nm and 588.997 nm correspond to transitions from ground state to two excited energy levels.

Values of T = 5800 K and P_e = 1 N m^{-2} were used to construct the curve of growth in $\log_{10}(W/\lambda)$ vs $\log_{10}[f N_a (\lambda/500 \text{ nm})]$:



The two Na I lines' wavelengths, equivalent widths in the solar spectrum, and oscillator strengths are given in the table below:

λ (nm)	W (nm)	f	$\log_{10}(W/\lambda)$	$\log_{10}[f(\lambda/500 \text{ nm})]$
330.238	0.0088	0.0214	−4.58	−1.85
588.997	0.0730	0.645	−3.90	−0.12

From the curve of growth, we can find the $\text{Log}_{10}[f N_a (\lambda/500 \text{ nm})]$ values for the observed $\log_{10}(W/\lambda)$.

$$\log_{10}\left(\frac{f N_a \lambda}{500 \text{ nm}}\right) = 17.20 \quad \text{for the 330.238 nm line}$$

$$= 18.83 \quad \text{for the 588.997 nm line.}$$

$$\log_{10} N_a = \log_{10}\left(\frac{f N_a \lambda}{500\ \text{nm}}\right) - \log_{10}\left(\frac{f\lambda}{500\ \text{nm}}\right),$$

$$\log_{10} N_a = 17.15 - (-1.85) = 19.00 \quad \text{for the 330.238 nm line}$$

$$\log_{10} N_a = 18.80 - (-0.12) = 18.92 \quad \text{for the 588.997 nm line.}$$

The column density of ground-state Na I is ~ 10^{19} NaI m^{-2}

How about Na I in excited states?

$$e^{-(E_b-E_a)/kT} = e^{-hc/\lambda kT}$$

$$= 5.45 \times 10^{-4} \quad \text{for the 330.238 nm line}$$

$$= 1.48 \times 10^{-2} \quad \text{for the 588.997 nm line,}$$

Good, we don't need to worry about excited Na I.

How about Na II?

Use Saha equation to evaluate the ratio of NaII/NaI.

$$\frac{N_{II}}{N_I} = \frac{2kT Z_{II}}{P_e Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

$Z_I = 2.4$, $Z_{II} = 1.0$, $\chi_I = 5.14$ eV ➔ $N_{II}/N_I = 2.43\times10^3$.

Therefore, $N = 2430 N_I = 2.43\times10^{22}\ m^{-2}$.

The mass of a Na atom is 3.82×10^{-26} kg, so the mass of Na above the photosphere is roughly 9.3×10^{-4} kg m^{-2}. This can be compared with the mass of H, 11 kg m^{-2}.

Elemental abundances in the solar atmosphere have been determined. They are expressed as relative abundances by $\log_{10}(N_{el}/N_H)+12$.

Element	Atomic Number	Log Relative Abundance
Hydrogen	1	12.00
Helium	2	10.93 ± 0.004
Oxygen	8	8.83 ± 0.06
Carbon	6	8.52 ± 0.06
Neon	10	8.08 ± 0.06
Nitrogen	7	7.92 ± 0.06
Magnesium	12	7.58 ± 0.05
Silicon	14	7.55 ± 0.05
Iron	26	7.50 ± 0.05
Sulfur	16	7.33 ± 0.11
Aluminum	13	6.47 ± 0.07
Argon	18	6.40 ± 0.06
Calcium	20	6.36 ± 0.02
Sodium	11	6.33 ± 0.03
Nickel	28	6.25 ± 0.04