Astronomy 404
September 23, 2013

Chapter 9. Stellar Atmospheres

Opacity (absorption coefficient) κ_λ :

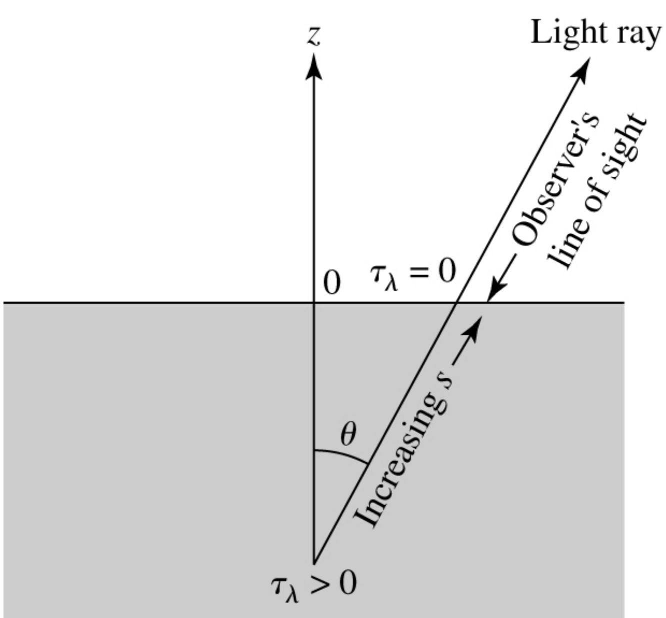
$$dI_\lambda = -\kappa_\lambda \rho I_\lambda \, ds.$$

Mean free path:

$$\ell = \frac{1}{\kappa_\lambda \rho} = \frac{1}{n\sigma_\lambda}$$

Optical depth τ_λ :

$$d\tau_\lambda = -\kappa_\lambda \rho \, ds,$$



$$I_\lambda = I_{\lambda,0} e^{-\tau_\lambda}$$

General Sources of Opacity:

1. Bound-bound transitions
2. Bound-free absorption (→ Balmer jump)
3. Free-free absorption
4. Electron scattering (Thompson scattering)

$$\sigma_T = \frac{1}{6\pi\epsilon_0^2}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-29}\ \mathrm{m}^2$$

Continuum opacity due to H^-

$$\mathrm{H} + e^- \rightleftharpoons \mathrm{H}^- + \gamma$$

The binding energy of H^- is 0.754 eV (photon energy for λ=1640nm). H^- ions are an important source of continuum opacity for stars cooler than F0.

For B, A stars, photoionization of hydrogen and free-free absorption are the main source of continuum opacity.

For O stars, electron scattering and photoionization of He are important for continuum opacity.

For really cool stars, bound-bound and bound-free transitions and photodissociation of molecules become important.

The total opacity is the sum of opacities due to all these sources:

$$\kappa_\lambda = \kappa_{\lambda,\mathrm{bb}} + \kappa_{\lambda,\mathrm{bf}} + \kappa_{\lambda,\mathrm{ff}} + \kappa_{\mathrm{es}} + \kappa_{\mathrm{H}^-}$$

The total opacity depends not only on wavelength of the light being considered but also on the composition, density, and temperature of the stellar material.

The Rosseland Mean Opacity

$$\frac{1}{\overline{\kappa}} \equiv \frac{\displaystyle\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T}\, d\nu}{\displaystyle\int_0^\infty \frac{\partial B_\nu(T)}{\partial T}\, d\nu}.$$

There are no simple equations to describe all of the complex contributions to the opacities due to bound-bound transitions.

Approximations of bound-free and free-free opacities:

$$\overline{\kappa}_{\rm bf} = 4.34 \times 10^{21} \frac{g_{\rm bf}}{t} Z(1+X) \frac{\rho}{T^{3.5}} \ {\rm m^2\ kg^{-1}}$$

$$\overline{\kappa}_{\rm ff} = 3.68 \times 10^{18}\, g_{\rm ff}\, (1-Z)(1+X) \frac{\rho}{T^{3.5}} \ {\rm m^2\ kg^{-1}}$$

where X ≡ (total mass of H) / (total mass of gas)
Y ≡ (total mass of He) / (total mass of gas)
Z ≡ (total mass of metals) / (total mass of gas)
X + Y + Z = 1.

The Gaunt factors g_{ff-} and g_{bf} are both ~ 1 for the visible and UV wavelengths of interest in stellar atmospheres.

The correction factor t in $\kappa_{\rm bf}$ is the **guillotine factor** that describes the cutoff of an atom's contribution to the opacity after it is ionized. Typical values of t lie between 1 and 100.

Both bf and ff opacities follow **Kramers opacity law**:

$$\overline{\kappa} = \kappa_0 \rho / T^{3.5}$$

The cross section for electron scattering is independent of wavelengths, so the opacity is:

$$\bar{\kappa}_{es} = 0.02(1 + X)\ \mathrm{m^2\ kg^{-1}}$$

The mean opacity provided by H^- ions can be included for temperatures 3000-6000 K and densities between
10^{-7} and 10^{-2} kg m^{-3} when X ~ 0.7 and 0.001< Z <0.03
(typical values for main sequence stars):

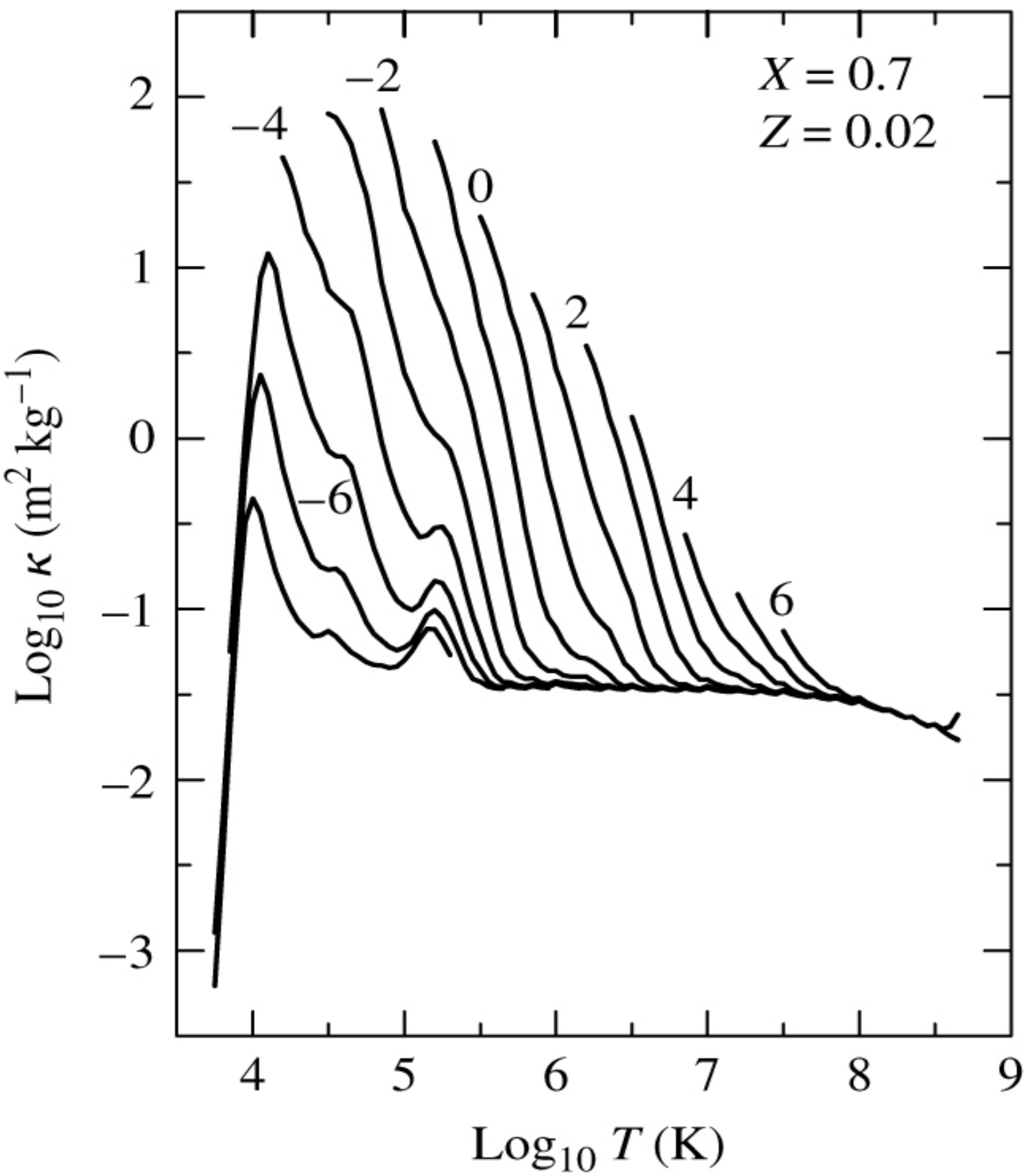
$$\bar{\kappa}_{H^-} \approx 7.9 \times 10^{-34}(Z/0.02)\rho^{1/2}T^9\ \mathrm{m^2\ kg^{-1}}$$

The total Rosseland mean opacity is:

$$\bar{\kappa} = \overline{\kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es} + \kappa_{H^-}}$$

The Rosseland mean opacity has been calculated by Carlos Iglesias and Forrest Rogers for X=0.7 and Z=0.02 as a function of temperature for different densities (labeled in log g in kg m^{-3}).

- At a given temperature, opacity increases with increasing density.
- At T < 10^4 K, opacity increases with increasing T (ionizing H & He)
- At T > 10^4 K, H ionized, bf and ff opacities follows Kramers law $\propto T^{-3.5}$
- Bump at 40,000 K is caused by the ionization of H II.
- Bump at 10^5 K is caused by the ionization of metals, most notably Fe.
- At very high temperatures, all ionized, electron scattering dominates.



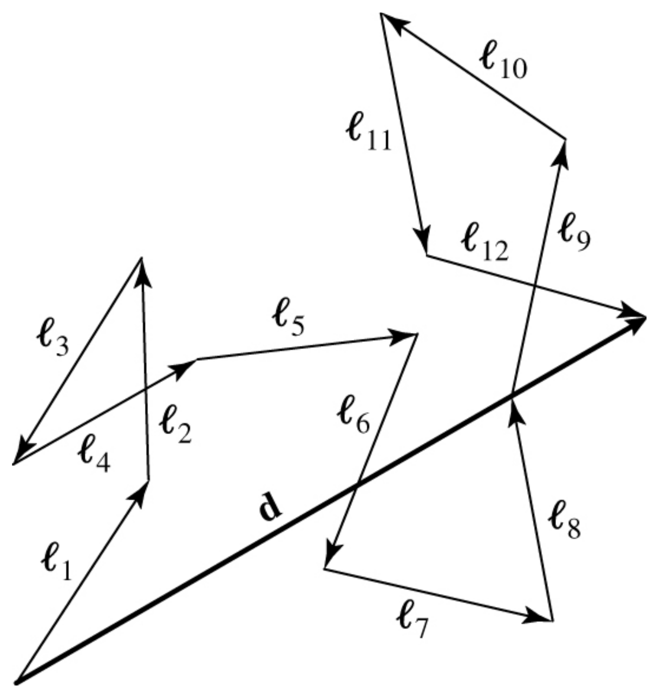
$$\overline{\kappa}_{\rm es} = 0.02(1 + X)\ {\rm m}^2\ {\rm kg}^{-1}$$

X = 0.7 => κ = 0.034, log κ = -1.47

Radiative Transfer

As light travels, both absorption and emission take place. The inverse of the absorption processes produce photons: bound-bound, free-bound, free-free,and electron scattering.

Photons do not travel outward directly. They are absorbed, emitted, and scattered. They diffuse upward through the stellar material, following a **random walk**.



$$\mathbf{d} = \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2 + \boldsymbol{\ell}_3 + \cdots + \boldsymbol{\ell}_N$$

$$\begin{aligned}\mathbf{d}\cdot\mathbf{d} &= \boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_1 + \boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_2 + \cdots + \boldsymbol{\ell}_1\cdot\boldsymbol{\ell}_N \\ &\quad + \boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_1 + \boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_2 + \cdots + \boldsymbol{\ell}_2\cdot\boldsymbol{\ell}_N \\ &\quad + \cdots + \boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_1 + \boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_2 + \cdots + \boldsymbol{\ell}_N\cdot\boldsymbol{\ell}_N \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\boldsymbol{\ell}_i\cdot\boldsymbol{\ell}_j,\end{aligned}$$

$$d^2 = N\ell^2 + \ell^2[\cos\theta_{12} + \cos\theta_{13} + \cdots + \cos\theta_{1N}$$

$$+ \cos\theta_{21} + \cos\theta_{23} + \cdots + \cos\theta_{2N}$$

$$+ \cdots + \cos\theta_{N1} + \cos\theta_{N2} + \cdots + \cos\theta_{N(N-1)}]$$

$$= N\ell^2 + \ell^2 \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \cos\theta_{ij},$$

The sum cos of random angles is 0. Therefore,

$$d = \ell\sqrt{N}$$

As the optical depthτ_λ is the number of mean free paths, $d = \tau_\lambda \ell$.

$$d = \tau_\lambda \ell = \ell\sqrt{N}$$

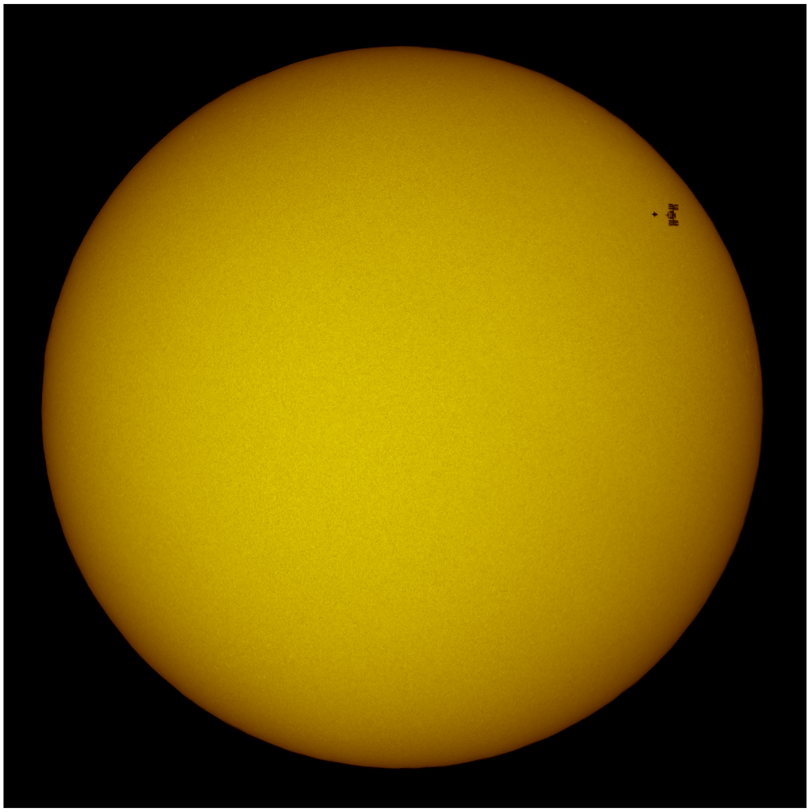
The average number of steps needed for a photon to travel the distance before leaving the surface is then $N = \tau_\lambda^2$, for $\tau_\lambda \gg 1$.

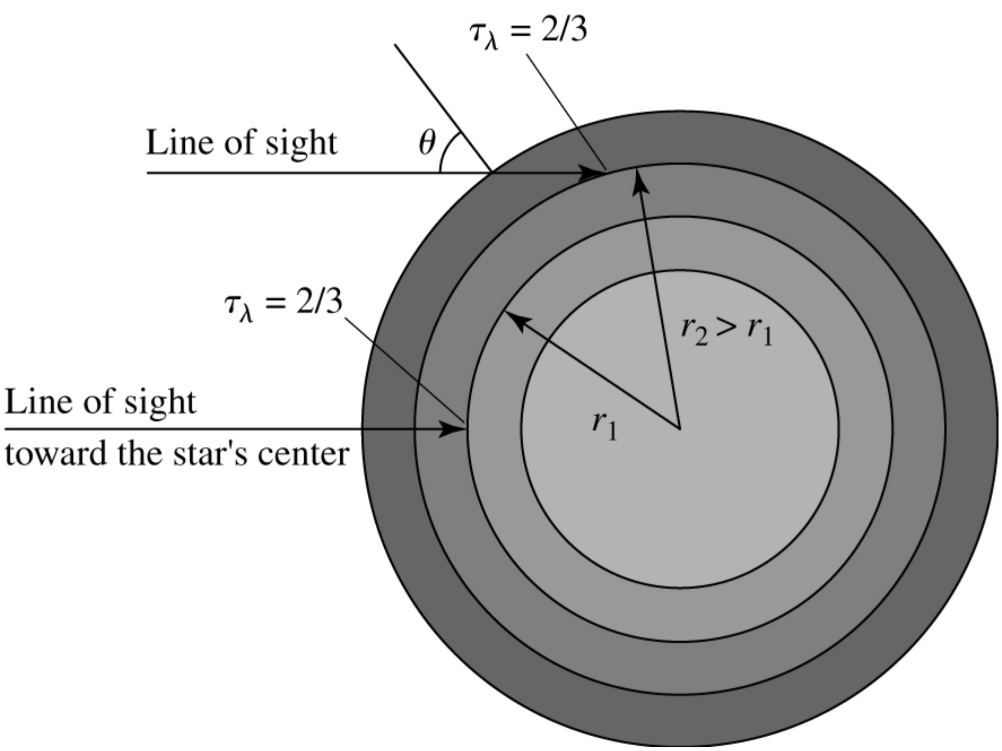
When $\tau_\lambda \sim 1$, a photon may escape from the star.

It is more likely that photons escape from $\tau_\lambda = 2/3$. Looking into a star at any angle, we always look back to $\tau_\lambda = 2/3$, where the *photosphere* is.

As temperature decreases outward, the (blackbody) radiation from the photosphere travels through cooler material, and if the line opacity is high, we see a shallower layer for the line. The temperature is lower, so the intensity is lower, resulting in an absorption line. Kircchoff's 3rd law.

Limb darkening





Radiation pressure gradient causes the net movement of photons toward the surface that carries the radiative flux:

$$\frac{dP_{\rm rad}}{dr} = -\frac{\overline{\kappa}\rho}{c} F_{\rm rad}$$

The Radiative Transfer Equation

Emission increases the specific intensity:

$$dI_\lambda = j_\lambda \rho \, ds$$

where j_λ is the emission coefficient of the gas, in units of m s^{-3} sr^{-1}.

Now consider both the absorption and emission,

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda \, ds + j_\lambda \rho \, ds$$

Dividing both side by $-\kappa_\lambda \rho \, ds$, we get

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}$$

The equation of radiative transfer:

$$\boxed{-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda.}$$

where $S_\lambda \equiv j_\lambda / \kappa_\lambda$ is called the **source function**, in W m^{-3}sr^{-1}.