Astronomy 404
September 20, 2013

Chapter 9. Stellar Atmospheres

Description of Radiation Field

Specific intensity:

$$I_\lambda \equiv \frac{\partial I}{\partial \lambda} \equiv \frac{E_\lambda \, d\lambda}{d\lambda \, dt \, dA \, \cos\theta \, d\Omega}.$$

Specific energy density:

$$u_\lambda \, d\lambda = \frac{4\pi}{c} \langle I_\lambda \rangle \, d\lambda$$

Specific radiation flux:

$$F_\lambda \, d\lambda = \int I_\lambda \, d\lambda \, \cos\theta \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, d\lambda \, \cos\theta \, \sin\theta \, d\theta \, d\phi$$

Radiation pressure:

$$P_{\mathrm{rad},\lambda} \, d\lambda = \frac{1}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, d\lambda \, \cos^2\theta \, \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi}{3c} I_\lambda \, d\lambda \qquad \text{(isotropic radiation field)}$$

For isotropic radiation field, $P_{\mathrm{rad}} = u/3$.

Local Thermodynamic Equilibrium (LTE)

The temperature changes over the mean free paths of particles and photons are insignificant.

Let's see how LTE works in the Sun's atmosphere…

The temperature varies from 5580 K to 5790 K over 25.0 km.
The *temperature scale height*, H_T, is given by

$$H_T \equiv \frac{T}{|dT/dr|} = \frac{5685 \text{ K}}{(5790 \text{ K} - 5580 \text{ K})/(25.0 \text{ km})} = 677 \text{ km}$$

The density here is about $\rho = 2.1\times10^{-4}$ kg m^{-3}, consisting primarily of HI atoms in the ground state. The number density is then

$$n = \frac{\rho}{m_H} = 1.25 \times 10^{23} \text{ m}^{-3}$$

The collisional cross section of an HI atom is $\sigma = \pi\,(2a_0)^2$, where a_0 is the Bohr radius. [This is a classical approximation.]

Within time t, the cross section sweeps a volume σvt , and the number of collisions encountered is $n\sigma vt$. The average distance traveled between collisions is the *mean free path* :

$$\ell = \frac{vt}{n\sigma vt} = \frac{1}{n\sigma}$$

For HI, a_0 is 5.29×10^{-11} m, and $\sigma = 3.52\times10^{-20}$ m^2, and the mean free path is 2.27×10^{-4} m.

The particle mean free path is << temperature scale height.
So, LTE is OK.

Next we consider photons…

Photons can be removed from a beam via **absorption**, which includes both true absorption and scattering.

The intensity of radiation changes as it travels through gas:

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda \, ds.$$

ρ is the density of the gas, κ_λ is the absorption coefficient or opacity, and ds is the distance traveled.

The opacity κ_λ is the cross section for absorbing photons at λ per unit mass of stellar material, so has the units of $m^2 \, kg^{-1}$.

Opacity depends on composition, density, and temperature.

Consider a beam of light travels through a gas from $s = 0$ to s .

$$\int_{I_{\lambda,0}}^{I_{\lambda,f}} \frac{dI_\lambda}{I_\lambda} = -\int_0^s \kappa_\lambda \rho \, ds.$$

$$I_\lambda = I_{\lambda,0} e^{-\int_0^s \kappa_\lambda \rho \, ds}$$

For a uniform gas of constant opacity and density:

$$I_\lambda = I_{\lambda,0} e^{-\kappa_\lambda \rho s}$$

The e-folding characteristic distance is

$$\ell = 1/\kappa_\lambda \rho.$$

In the solar atmosphere, $\rho = 2.1\times10^{-4}$ kg m^{-3}, and the opacity at

500 nm is κ_{500} = 0.03 m^2 kg^{-1}. Thus,

$$\ell = \frac{1}{\kappa_{500}\rho} = 160 \text{ km}$$

Compared with the temperature scale height of 677 km, photons do not see LTE here...

Optical Depth

The mean free path of photons is:

$$\ell = \frac{1}{\kappa_\lambda \rho} = \frac{1}{n\sigma_\lambda}$$

where σ_λ can be considered the cross section of a photon.

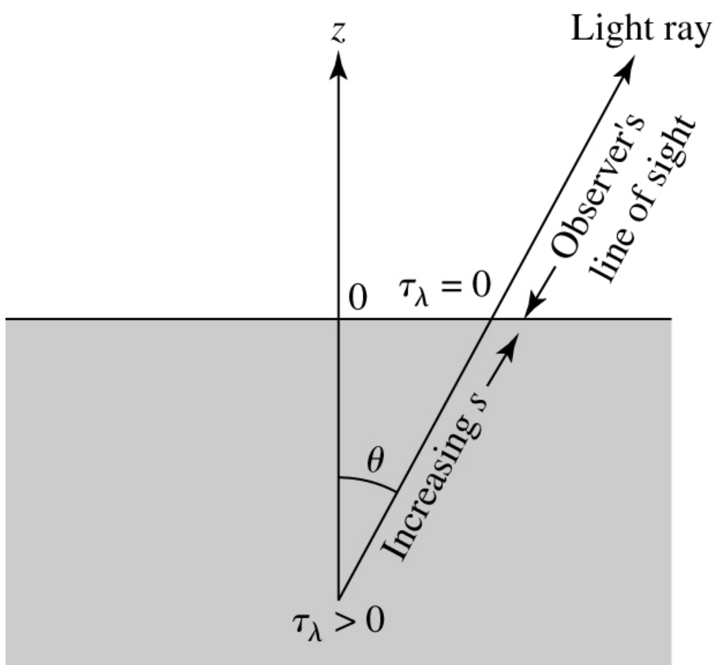
Optical depth τ_λ is defined as

$$\boxed{d\tau_\lambda = -\kappa_\lambda \rho \, ds,}$$

s is the distance measured along the photon's path (or looking back into the star).

From the initial position $s = 0$ to the final position s, the difference in the optical depth is:

$$\Delta\tau_\lambda = \tau_{\lambda,f} - \tau_{\lambda,0} = -\int_0^s \kappa_\lambda \rho \, ds$$



In the outermost layer of atmosphere, $\tau_\lambda = 0$.
The initial optical depth of a light ray that traveled a distance *s* to the top of the atmosphere is:

$$0 - \tau_{\lambda,0} = -\int_0^s \kappa_\lambda \rho \, ds$$

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho \, ds.$$

$$I_\lambda = I_{\lambda,0} e^{-\kappa_\lambda \rho s}$$

$$I_\lambda = I_{\lambda,0} e^{-\tau_\lambda}$$

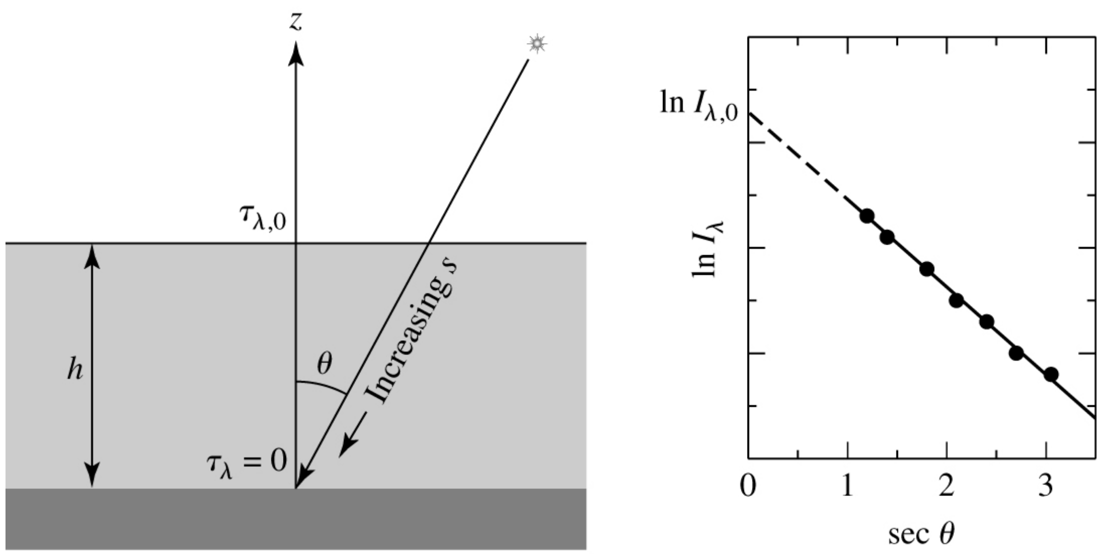
Optical depth can be thought of as the number of mean free paths from the original position to the surface, along the ray's path.
Typically, we see no deeper than $\tau_\lambda = 1$.

$\tau_\lambda >> 1$ optically thick ; $\tau_\lambda << 1$ optically thin

Atmosphere can be optical thick at some wavelengths and optically thick at other wavelengths.

For example, the Earth's atmosphere is optically thin at visible wavelengths, but optically thick at X-ray wavelengths.

We observe through the Earth's atmosphere, which absorbs starlight.



(a) (b)

$ds = -dz / \cos\theta = -\sec\theta\, dz$

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho\, ds = -\int_h^0 \kappa_\lambda \rho \frac{dz}{\cos\theta} = \sec\theta \int_0^h \kappa_\lambda \rho\, dz = \tau_{\lambda,0} \sec\theta,$$

where $\tau_{\lambda,0}$ is the optical depth at $\theta = 0$.

$$I_\lambda = I_{\lambda,0} e^{-\tau_{\lambda,0} \sec\theta}$$

Two unknowns, one equation. But, we can make many observations at different zenith angles θ, and make a plot to determine $I_{\lambda,0}$ and $\tau_{\lambda,0}$.

General Sources of Opacity

1. **Bound-bound transitions**
   No simple equation for opacity due to bound-bound transitions.

2. **Bound-free absorption**
   This is photoionization. The cross section for photoionizing an HI atom at quantum state n by a photon of λ is:

$$\sigma_{\rm bf} = 1.31 \times 10^{-19} \frac{1}{n^5} \left(\frac{\lambda}{500\ \rm nm}\right)^3 \ \rm m^2$$

3. **Free-free absorption**
   Free-free emission is known as bremsstrahlung.

4. **Electron scattering**
   This is Thompson scattering, i.e., the electric field of the incident wave accelerates the electron and causes it to radiate at the same frequency as the incident wave.

$$\sigma_T = \frac{1}{6\pi\epsilon_0^2}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65 \times 10^{-29}\ \rm m^2$$

Electron scattering is important only when the electron density is high, which requires high temperatures, such as early-type stars and stellar interiors.

Balmer jump

$E_2 = -13.6/2^2$ eV = -3.4 eV

Photons with energy greater than 3.4 eV can photoionize an HI at $n = 2$. $\lambda \leq 364.7$ nm.