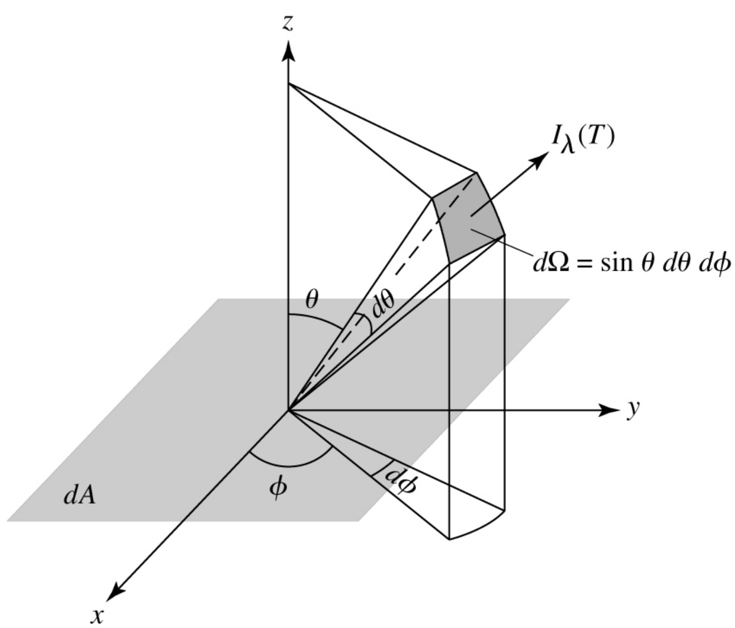
Astronomy 404
September 18, 2013

Chapter 9. Stellar Atmospheres

The energy produced in the interior of a star propagates outward. The continuum (blackbody) emission from a star is from the "photosphere", at the base of the atmosphere. As light travels through the atmosphere, photons get absorbed and re-emitted repeatedly until they leave the atmosphere. The stellar spectrum depends on the temperature, density, and composition of the stellar atmosphere.
(Any other physical properties affecting the spectrum?)

9.1 Description of the Radiation Field



The Specific and Mean Intensity

A ray of light with a wavelength between λ and λ+*d*λ, passing through a surface area *dA* at an angle θ into a solid angle $d\Omega$. Defining $E_\lambda \equiv \partial E/\partial\lambda$, $E_\lambda\, d\lambda$ is the amount of energy going into the cone in a time interval *dt*. Then

$$E_\lambda\, d\lambda = I_\lambda\, d\lambda\; dt\; dA\; \cos\theta\; d\Omega$$
$$= I_\lambda\, d\lambda\; dt\; dA\; \cos\theta\; \sin\theta\; d\theta\; d\phi \qquad \text{(units: J)}$$

***I_λ* is the specific intensity and has units of W m^{-3} sr^{-1}.**

The specific intensity is usually referred to simply as the *intensity*.

The Planck function B_λ is an example of specific intensity for the special case of blackbody radiation.

$$I_\lambda \equiv \frac{\partial I}{\partial \lambda} \equiv \frac{E_\lambda \, d\lambda}{d\lambda \, dt \, dA \, \cos\theta \, d\Omega}.$$

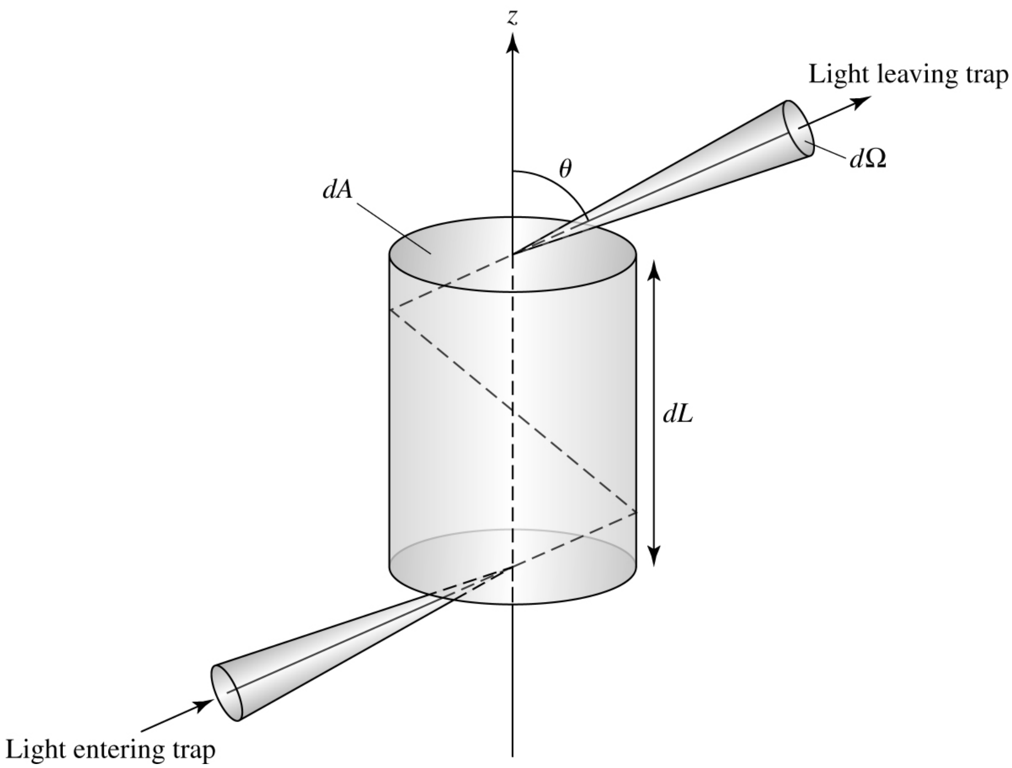
Imagine a light ray of intensity I_λ propagating through a vacuum (no material for it to interact with). As I_λ is defined in the limit of $d\Omega \rightarrow 0$, the energy of the ray does not spread out (or diverge) . The intensity is therefore constant along any ray traveling through empty space.

Generally, I_λ does vary with direction. The **mean intensity** of the radiation is:

$$\langle I_\lambda \rangle \equiv \frac{1}{4\pi} \int I_\lambda \, d\Omega = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, \sin\theta \, d\theta \, d\phi$$

For an isotropic radiation field, $\langle I_\lambda \rangle = I_\lambda$.
Blackbody radiation is isotropic and has $\langle I_\lambda \rangle = B_\lambda$.

Specific Energy Density



How much energy is "trapped" in this volume?
The amount of time for light to travel across is $dt = dL / (c \cos\theta)$.

$$E_\lambda \, d\lambda = I_\lambda \; d\lambda \; dt \; dA \; \cos\theta \; d\Omega = I_\lambda \; d\lambda \; dA \; d\Omega \; \frac{dL}{c}$$
$$= \frac{1}{c} I_\lambda \; d\lambda \, d\Omega \; dV$$

The specific energy density (energy per unit volume with a wavelength between λ and λ+$d\lambda$) is $u_\lambda \, d\lambda$:

$$u_\lambda \, d\lambda = \frac{1}{c} \int I_\lambda \, d\lambda \, d\Omega$$
$$= \frac{1}{c} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, d\lambda \, \sin\theta \, d\theta \, d\phi$$
$$= \frac{4\pi}{c} \langle I_\lambda \rangle \, d\lambda.$$

For an isotropic radiation field, $u_\lambda \, d\lambda = (4\pi/c) \, I_\lambda \; d\lambda$, and for blackbody radiation:

$$u_\lambda \, d\lambda = \frac{4\pi}{c} B_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

One can also express the blackbody energy density in terms of the frequency ν :

$$u_\nu \, d\nu = \frac{4\pi}{c} B_\nu \, d\nu = \frac{8\pi h\nu^3/c^3}{e^{h\nu/kT} - 1} \, d\nu.$$

The total energy density, u, is found by integrating over all λ or ν :

$$u = \int_0^\infty u_\lambda \, d\lambda = \int_0^\infty u_\nu \, d\nu.$$

For blackbody radiation, $I_\lambda = B_\lambda$, and

$$u = \frac{4\pi}{c} \int_0^\infty B_\lambda(T) \, d\lambda = \frac{4\sigma T^4}{c} = aT^4$$

where $a \equiv 4\sigma/c$ is known as the *radiation constant*, and has the value $a = 7.565767 \times 10^{-16}$ J m^{-3} K^{-4} .

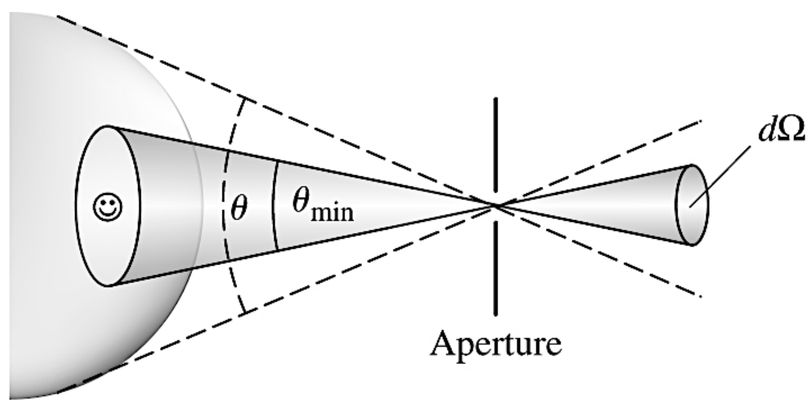
The Specific Radiation Flux

Specific radiation flux F_λ

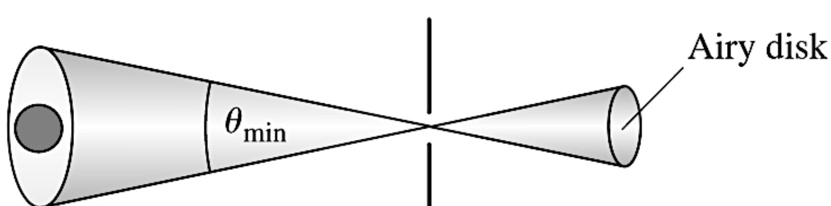
$F_\lambda \, d\lambda$ is the net energy having a wavelength between λ and λ+dλ that passes each second through a unit area in the direction of z :

$$F_\lambda \, d\lambda = \int I_\lambda \, d\lambda \cos\theta \, d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \, d\lambda \cos\theta \sin\theta \, d\theta \, d\phi$$

When we observe a celestial source, do we measure specific intensity of specific radiation flux?



(a)



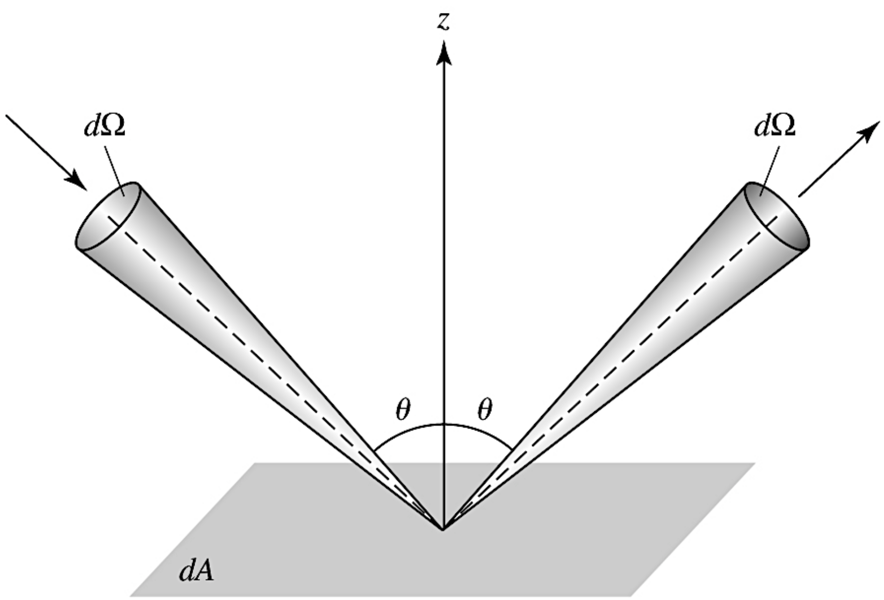
(b)

If an object is resolved, specific intensity is measured, and it is independent of distance.

If an object is not resolved, specific radiation flux is measured, and it follows the inverse square law, $\propto$ (distance)$^{-2}$.

Radiation Pressure

Photon has an energy $E = h\nu = hc/\lambda$,
and carries a momentum $p = E/c$.
As photons carry momemtum, they can exert pressure.



$$
\begin{aligned}
dp_\lambda\, d\lambda &= \left[(p_\lambda)_{\text{final},z} - (p_\lambda)_{\text{initial},z}\right] d\lambda \\
&= \left[\frac{E_\lambda \cos\theta}{c} - \left(-\frac{E_\lambda \cos\theta}{c}\right)\right] d\lambda \\
&= \frac{2\, E_\lambda \cos\theta}{c}\, d\lambda \\
&= \frac{2}{c}\, I_\lambda\, d\lambda\, dt\, dA\, \cos^2\theta\, d\Omega,
\end{aligned}
$$

(dp_λ / dt) / dA is the force per unit area with the solid angle $d\Omega$.

$$P_{\mathrm{rad},\lambda}\, d\lambda = \frac{2}{c}\int_{\text{hemisphere}} I_\lambda\, d\lambda\, \cos^2\theta\, d\Omega \qquad \text{(reflection)}$$

$$= \frac{2}{c}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I_\lambda\, d\lambda\, \cos^2\theta\, \sin\theta\, d\theta\, d\phi.$$

The above expression is for photons bouncing off a reflective surface. However, there is no "reflective surface" in stars, photons just stream through surface dA. Therefore, the leading factor of 2 should be removed:

$$P_{\mathrm{rad},\lambda}\, d\lambda = \frac{1}{c}\int_{\text{sphere}} I_\lambda\, d\lambda\, \cos^2\theta\, d\Omega \qquad \text{(transmission)}$$

$$= \frac{1}{c}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} I_\lambda\, d\lambda\, \cos^2\theta\, \sin\theta\, d\theta\, d\phi$$

$$= \frac{4\pi}{3c} I_\lambda\, d\lambda \qquad \text{(isotropic radiation field).}$$

The total radiation pressure (integrated over all wavelengths) is:

$$P_{\mathrm{rad}} = \int_0^\infty P_{\mathrm{rad},\lambda}\, d\lambda$$

For blackbody radiation:

$$P_{\mathrm{rad}} = \frac{4\pi}{3c}\int_0^\infty B_\lambda(T)\, d\lambda = \frac{4\sigma T^4}{3c} = \frac{1}{3}aT^4 = \frac{1}{3}u$$

The blackbody radiation pressure is 1/3 the energy density.
The pressure for an ideal monatomic gas is 2/3 the energy density.

Temperature and Local Thermodynamic Equilibrium

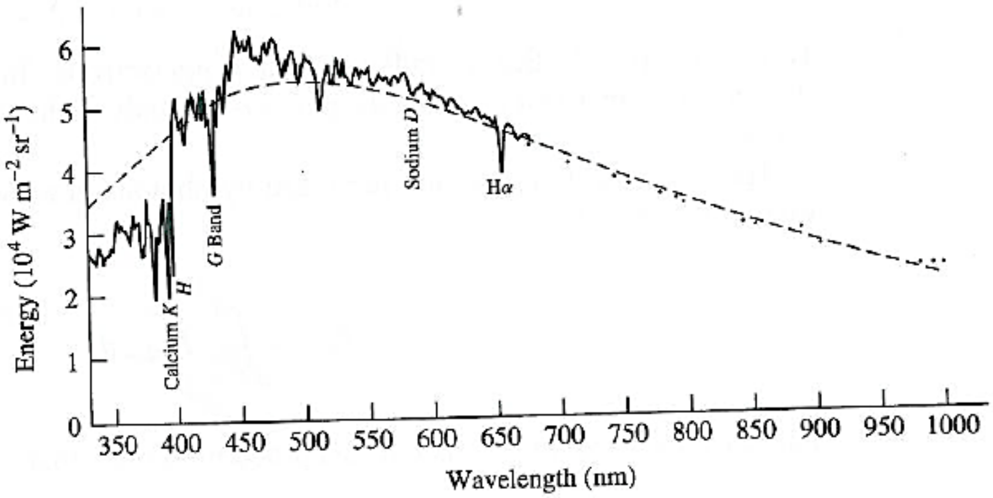


FIGURE 9.5 The spectrum of the Sun in 2 nm wavelength intervals. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure adapted from Aller, *Atoms, Stars, and Nebulae*, Third Edition, Cambridge University Press, New York, 1991.)

The spectrum of the Sun is far from a blackbody curve!

Line Blanketing!

Effective temperature – obtained from $L = 4\pi R^2 \sigma T^4$

Excitation temperature – defined by the Boltzmann equation

Ionization temperature – defined by the Saha equation

Kinetic temperature – in the Maxwell-Boltzmann vel distribution

Color temperature – fitting stellar continuum by Planck function

These temperatures are the same at thermodynamic equilibrium, under which condition energy does not flow in or out the "box". Energy does flow from stellar interior to the surface.
However, the temperature does not change significantly over the distance traveled by photons and particles between collisions
➔ **local thermodynamic equilibrium (LTE)**