

Size distribution of Supernova remnants and the interstellar medium: the case of M33

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Aim, Motivation

- Supernova Remnants (SNRs) are fundamental to our understanding of the interstellar medium (ISM). The size distribution of SNRs can help explore various aspects of their evolution and interaction with the ISM.
- The form of the size distribution is determined mainly by:
 - 1) SN explosion energies
 - 2) the density and total pressure of the ISM

Method

- We have modeled the SNR size distributions and compared them with the observed ones.
- Since the observed sets of SNRs are the collection of objects with very different ages and diameters and evolving in different conditions of the ISM and with different initial conditions the statistical method of Monte Carlo can be used in modeling their statistical distributions.

Expansion of the remnant

- The problem of expansion of the supernova blast wave in general is well understood and described in number of detailed studies (e.g., [Ze'ldovich and Raizer 1968](#); [Bisnovatyi-Kogan and Silich 1995](#); [Ostriker and McKee 1988](#)). In the course of evolution with age, supernova remnant goes through several phases.
- The first, free expansion phase** lasts up to the moment when the swept-up mass equals the ejecta mass. For this stage of expansion we use the expression:

$$v_s = \frac{v_{0s}}{\sqrt{1 + (R_s / R_0)^3}}$$

- where $R_0 = \left(\frac{3M_{ej}}{4\pi\rho_0} \right)^{1/3}$, and $v_0 = \sqrt{\frac{2E_{0k}}{M_{ej}}}$

- This equation follows from the condition of kinetic energy conservation

$$\left(M_{ej} + \frac{4}{3}\pi\rho_0 R_s^3 \right) \cdot \left(\frac{dR_s}{dt} \right)^2 = 2E_{0k}$$

- and describes the smooth transition from the free expansion to the following stage of evolution.
- Since the duration of this phase is relatively short, less than ~ 1000 years, this period of life of a remnant plays only a minor role in the overall statistics of SNRs.

Expansion of the remnant

- By the time the swept up mass equals the ejecta mass a smooth transition from free – expansion to the adiabatic **Sedov-Taylor phase** begins (Truelove and McKee 1999). The radius R_s of the spherical blast wave, evolving in the homogeneous and stationary ISM, changes with time t according to the law

$$R_s = \left(\frac{\xi_0 E_0}{\rho_0} \right)^{1/5} t^{2/5}$$

- where E_0 - full (thermal plus kinetic) energy of the blast wave, ρ_0 - the mass density in the ISM, ξ_0 - dimensionless numerical constant.
- - The applicability of the Sedov solution to real SNR is determined by two main conditions that 1) the pressure of the ambient medium is negligible and 2) the condition of adiabaticity of matter inside the remnant is retained. Violation of any of these conditions leads to a violation of the applicability of the Sedov solution. In very tenuous environments the adiabaticity is retained up to the very large diameters at which the pressure of the interstellar medium becomes important.
- Important: the Sedov law of motion gives relatively good results up to the values of the shock Mach numbers of 2, though the internal structure of the shell begins to deviate from the Sedov self-similar solution much earlier, when $M_s < 6-10$ [Sedov,1972; Cox & Anderson,1980].

Expansion of the remnant

- When the temperature of the post-shock gas drops to $\sim 10^5$ K the SNR evolving in relatively dense medium enters **radiative (or pressure-driven snow plow) phase** of evolution. The time and radius at which this phase begins are [Cioffi et al. (1988)]

- $$t_{sf} = 3.61 \times 10^4 \zeta_m^{-5/14} E_{51}^{3/14} n_0^{-4/7} \text{ yr} \quad R_{sf} = 13.38 \left(\frac{E_{51}}{\mu n_0} \right)^{0.2} t_{sf,4}^{0.4} \text{ pc}$$

- The radial expansion of SNR is governed by

- mass
$$\frac{dM}{dt} = 4\pi \rho_0 R_s^2 v_s$$

- momentum
$$\frac{d(Mv_s)}{dt} = 4\pi R_s^2 (P_{in} - P_0)$$

- energy
$$\frac{dP_{in}}{dt} = -3\gamma P_{in} \cdot \frac{v_s}{R_s}$$

- conservation equations, where $M = \frac{4\pi}{3} \rho_0 R_s^3$ is the mass accumulated in the thin shell, $v_s = \frac{dR_s}{dt}$ is the velocity of the shell = shock velocity; P_{in} and P_0 - the pressure inside the shell and the total pressure in the ISM.

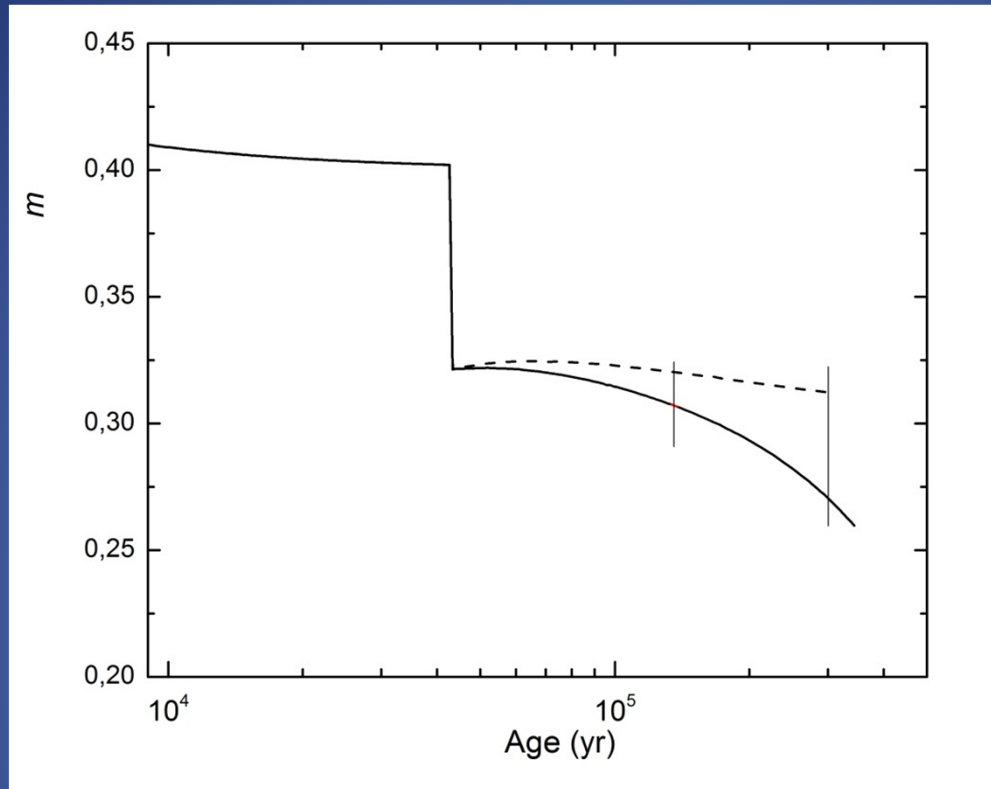
Expansion of the remnant

- The above system of equations can be reduced to one equation expressing the dependence of the shock Mach number on the radius of SNR

$$\gamma M_s^2 = \frac{2}{2-\gamma} \frac{P_{lin}}{P_0} \cdot \frac{R_l^{3\gamma}}{R_s^{3\gamma}} + \frac{C}{R_s^6} - 1$$

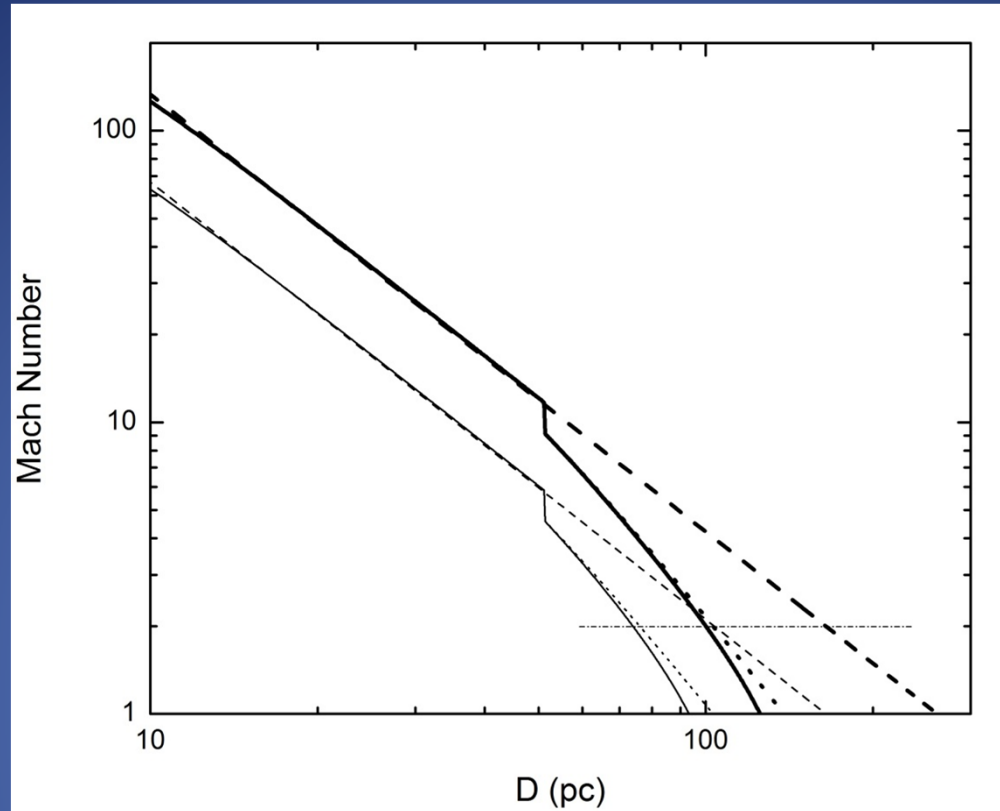
- Mach number is determined as $M_s = v_s / c_{ms0}$, where $c_{ms0} \equiv \sqrt{\gamma P_0 / \rho_0}$ is the (magneto-) sound speed in the ISM, C - integration constant.
- The dependences $M_s(R_s)$ and $v_s(R_s)$ can be numerically converted to time dependences with the help of integrals

$$t - t_{sf} = \int_{R_{sf}}^R \frac{dR'}{v(R')} = \frac{R_{sf}}{c_{s0}} \int_1^{R/R_{sf}} \frac{dx}{M_s(x)}$$



The dependence

of $m \equiv \frac{M_s \cdot c_{s0} t}{R_s}$ (assuming the expansion law as $R_s \propto t^m$,) on time. Solid line corresponds to the case of evolution with the effect of pressure of the ISM, dashed line – without. Vertical lines denote the moments when shock Mach number $M_s = 2$ (left) and $= 1$ (right). $E_0 = 10^{51}$ erg, $n_0 = 0.5 \text{ cm}^{-3}$.



The dependence of the Mach number on the diameter of SNR for two values of pressure: $P_0 = 1.17 \times 10^4 \text{ K cm}^{-3}$ (thick lines) and $4.64 \times 10^4 \text{ K cm}^{-3}$ (thin lines). The solid and dotted curves describe the snow plow solution with and without the pressure of the ISM respectively, and the dashed curves describe the Sedov solution without radiative cooling. The horizontal dash-dotted line corresponds to point $M_s = 2$. $E_0 = 10^{51} \text{ erg}$, $n_0 = 0.5 \text{ cm}^{-3}$.

The basic point is that, we cannot determine the final age and maximum size of SNR without taking the pressure of the ISM into account .

We assume that the active life of a SNR will end when its Mach number reaches the value of 2, and we adopt the diameter and age of the remnant at this moment as the maximum size and lifetime of SNR.

Observations show that SNRs have very different characteristics. This is partly due to the dispersion of the properties of SNe (mainly, SN energies in case of extended sources), partly due to the large spread of properties of the ISM – density, pressure, chemical abundances of the ISM, etc.

Model:

Every ΔT years a SN occurs, the initial parameters of which distributed randomly in intervals:

- the energy (kinetic) of the explosion $E_0 = [.2 ; 5] \times 10^{51} \text{ erg}$
- the mass of the ejecta $M_{ej} = [1; 5] \text{ Msun.}$

Parameters of the ISM, the total pressure and the density, where the SN is born, also are chosen in random manner:

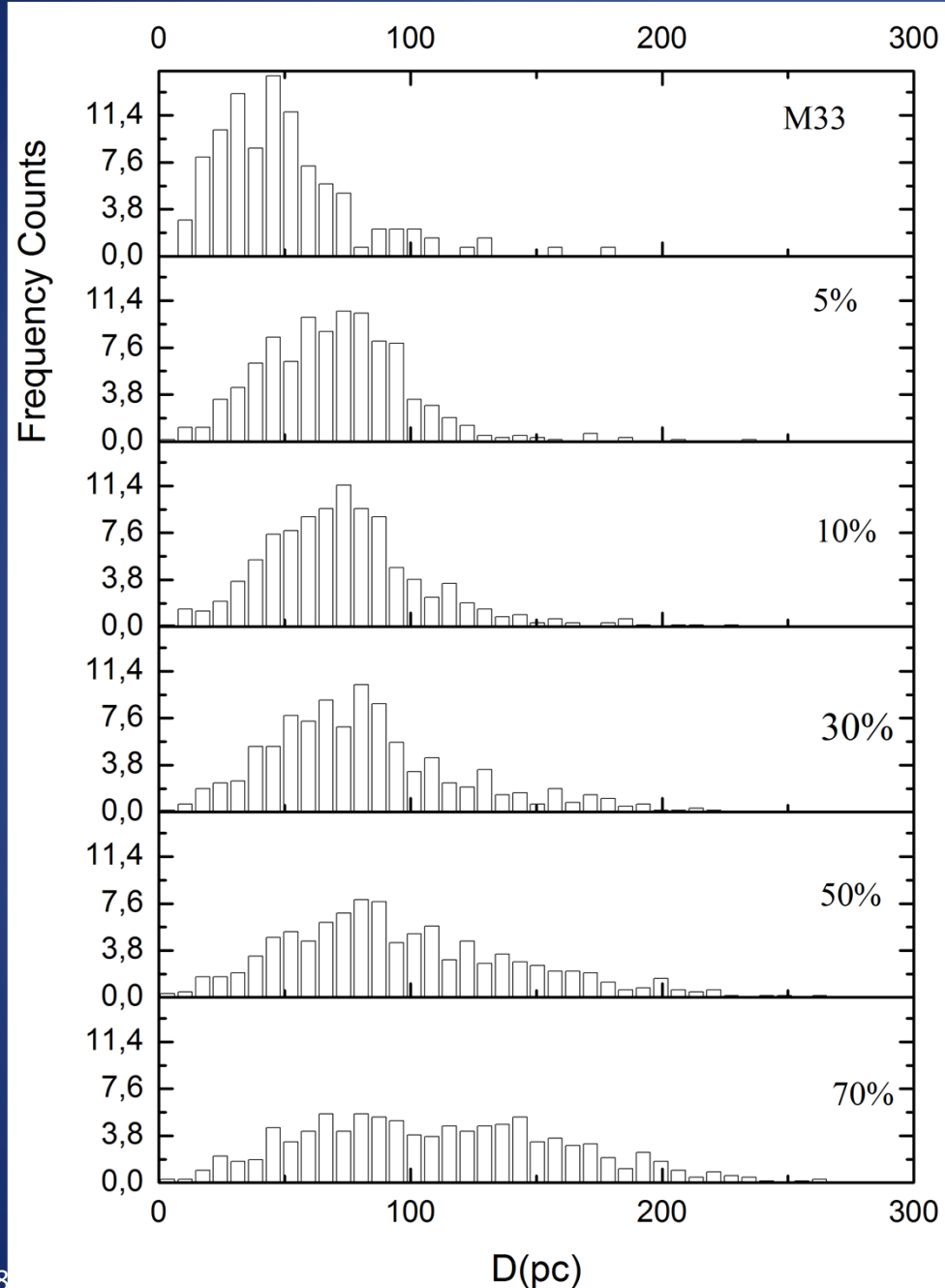
- the total pressure in the range $P_0 = [1; 5] \times 10^4 \text{ K cm}^{-3}$,

and for the density we choose three-phase model in which :

- hot, $n_h = [0.005; 0.1) \text{ cm}^{-3}$ ϕ_h
- warm $n_w = [0.1; 1) \text{ cm}^{-3}$ ϕ_w
- cold, $n_c = [1; 10] \text{ cm}^{-3}$ $(\phi_c \sim 0.01)$

ϕ_h (or ϕ_w) and ΔT - parameters TBD

We performed Monte-Carlo simulations where we varied ϕ_h as input parameter



The size distribution of 137 SNRs and SNR candidates in M33 (Long et al., 2010 ApJS,187,495) and SNRs Monte Carlo simulated for various values of the hot phase filling-factor. In simulations $\Delta T=250$ yr, the lifetime is determined as $t=t(M_s=2)$.

- In this figure the modeled SNRs are free from any selection effects, but observed SNRs in M33 are the ones detected in optical wavebands. Unfortunately, the evolution of optical luminosity of SNRs is poorly understood, that is why we cannot model this emission theoretically. Also, we don't know how the set of observational data used in figure are far from completeness.
- The other two channels by which we receive the observational information from SNRs are the radio and X-ray wavebands
- To compare the statistics of modeled SNRs with the observations we need two things:
 - a subset of observed in some waveband SNRs for which the statistical completeness is safely guaranteed
 - the reliable model of evolution of the emission in this same waveband.

Observational data: Radio

- Advantages – much more reliable observational data : the radio telescopes are sited on Earth permanently and it is possible to repeat the observations
- Disadvantages – the origin of the radio emission is not well understood, so problems arise in modeling the radio emission

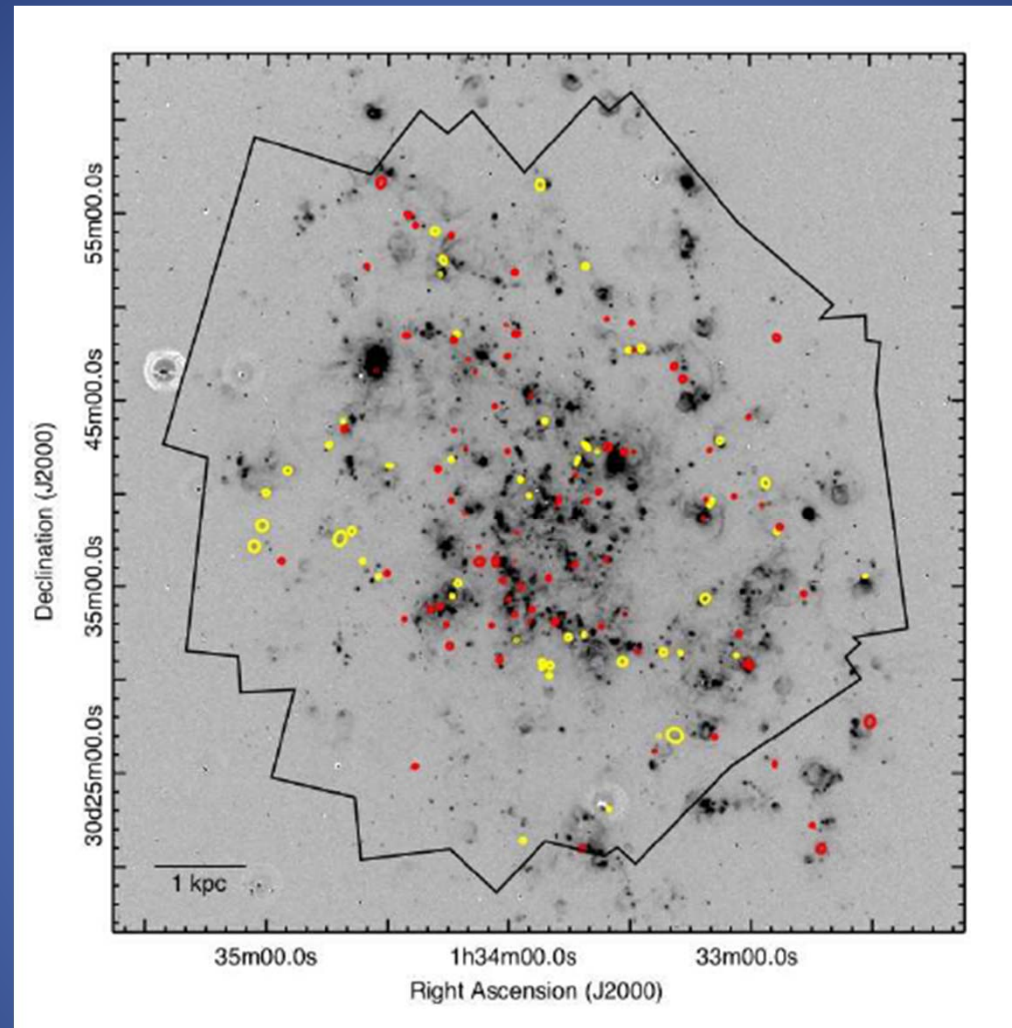
Observational data: X-Rays

- Advantages – practically all the evolved SNRs show X-ray emission of **thermal** nature, i.e. it is easy to model the evolution of the emission in this waveband.
- Disadvantages – In X-rays it is difficult to collect uniform statistical material because the X-ray telescopes have to be sent into space, and they have relatively short lifetimes and the observations have one-time character [it is impossible to “... *step twice into the same stream*”].
 - **Fortunately, recent work by Long et al (2010) (L10) is an exception**

(L10) - Long, K. S.; Blair, W. P.; Winkler, P. F.; Becker, R. H.; Gaetz, T.J.; Ghavamian, P.; Helfand, D.J.; Hughes, J.P.; Kirshner, R.P.; Kuntz, K.D.; McNeil, E.K.; Pannuti, T.G.; Plucinsky, P.P.; Saul, D.; Tüllmann, R.; Williams, B.
-The Chandra ACIS Survey of M33: X-ray, Optical, and Radio Properties of the Supernova Remnants,
ApJS, 187,495-559 (2010)

137 – optical SNRs and SNR candidates,
82 - of them detected in X-rays (0.35 - 8 keV)

The best collection of SNRs to date detected in X-ray wavelengths in any galaxy



Survey cover the inner region of M33 to a radius of about 18', or
4.3 kpc (d= 817 pc)

8/30/2012

Asvarov A.I.
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(T11) - Tüllmann, R.; Gaetz, T. J.; Plucinsky, P. P.; Kuntz, K. D.; Williams, B. F.; Pietsch, W.; Haberl, F.; Long, K. S.; Blair, W. P.; Sasaki, M.; Winkler, P. F.; Challis, P.; Pannuti, T. G.; Edgar, R. J.; Helfand, D. J.; Hughes, J. P.; Kirshner, R. P.; Mazeh, T.; Shporer, A.

-The Chandra ACIS Survey of M33 (ChASeM33): The Final Source Catalog

ApJS, 193, 31 (2011)

44 SNRs as point like sources

X-Ray emission of SNRs

$$\varepsilon_{ff} = \frac{2^5 \pi e^6}{3 m_e c^3} \left(\frac{2\pi}{3 k m_e} \right)^{1/2} g_{ff}(T_e) n_e \sum_i n_i Z_i^2 \cdot T_e^{-1/2} \exp\left(-\frac{E_x}{k T_e}\right) \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1}$$

- (Vink J., 2011)
- Assuming complete equilibration between ions and electrons the X-ray temperature and the shock velocity connected

$$v_s = (\gamma + 1) \left(\frac{k T_s}{2(\gamma - 1) m_p \mu_s} \right)^{1/2} = 715 (T_{\text{keV}} / \mu_s)^{1/2} \text{ km s}^{-1}$$

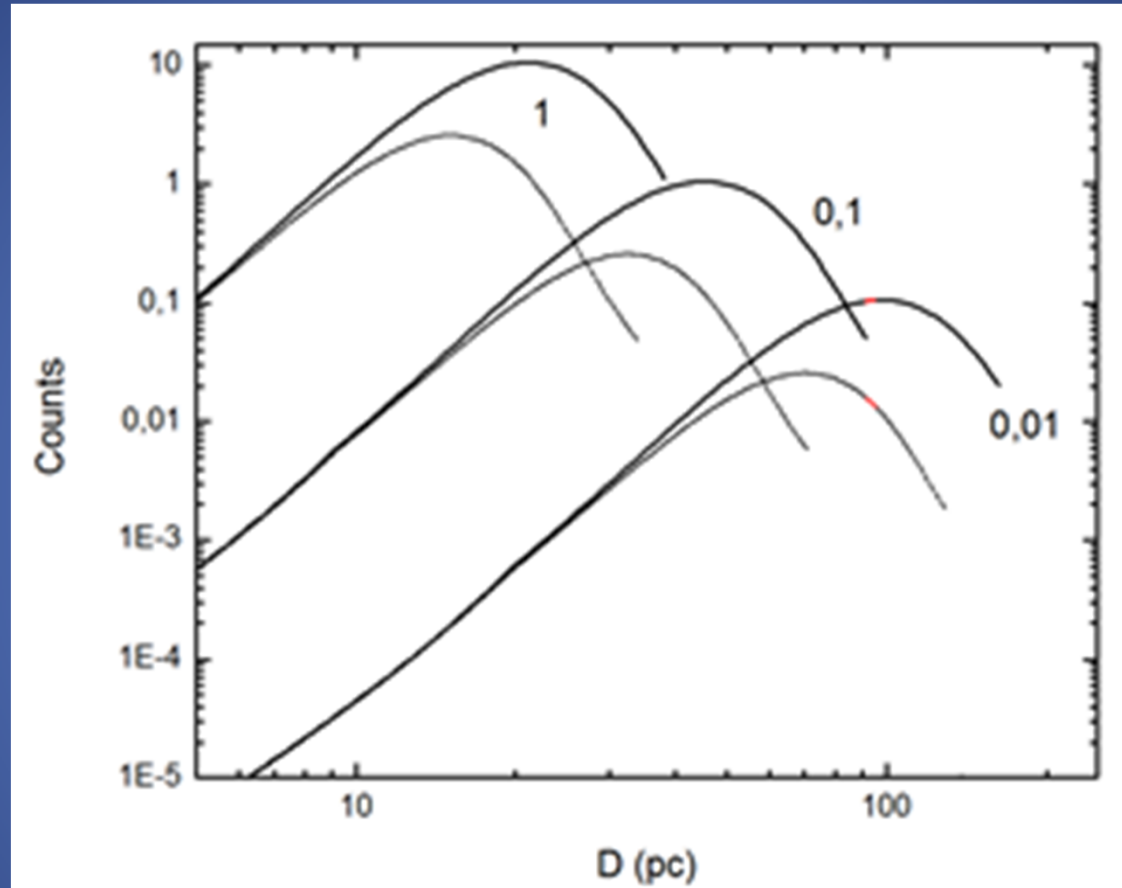
$$\Phi(E_{x1}; E_{x2}) = A \cdot \int_V dV \int_{E_{x1}}^{E_{x2}} dE_x n_e \sum_i n_i Z_i^2 \cdot T_e^{-1/2} E_x^{-1} \exp\left(-\frac{E_x}{k T_e} - \sigma(E_x) \cdot N_H\right) s^{-1}$$

X-Ray emission of SNRs

$$\Phi(E_{x1}; E_{x2}) = A \cdot \int_V dV \int_{E_{x1}}^{E_{x2}} dE_x n_e \sum_i n_i Z_i^2 \cdot T_e^{-1/2} E_x^{-1} \exp\left(-\frac{E_x}{kT_e} - \sigma(E_x) \cdot N_H\right) s^{-1}$$

- In calculations used:
- $\sigma(E_x) = (c_0 + c_1 E_x + c_2 E_x^2) E_x^{-3} \times 10^{-24} \text{ cm}^2$ (Zombek, 2007)
- $N_H = 10^{21} \text{ cm}^{-2}; \quad d = 817 \text{ kpc}$ L10
- $M_s \geq 10$ Sedov's self-similar solution
- $M_s < 10$ Cox & Anderson (1982) approximation

X-Ray emission of SNRs



The evolution of the count rates of X-ray photons in [0.36-1.1] (thick curves) and (1.1 – 2.6] (thin curve) with D (arbitrary normalization). For $n_0=0.01, 0.1$ and 1 cm^{-3} , $E_0=10^{51} \text{ erg}$, $N_H=10^{21} \text{ cm}^{-2}$, $d=817 \text{ kpc}$.

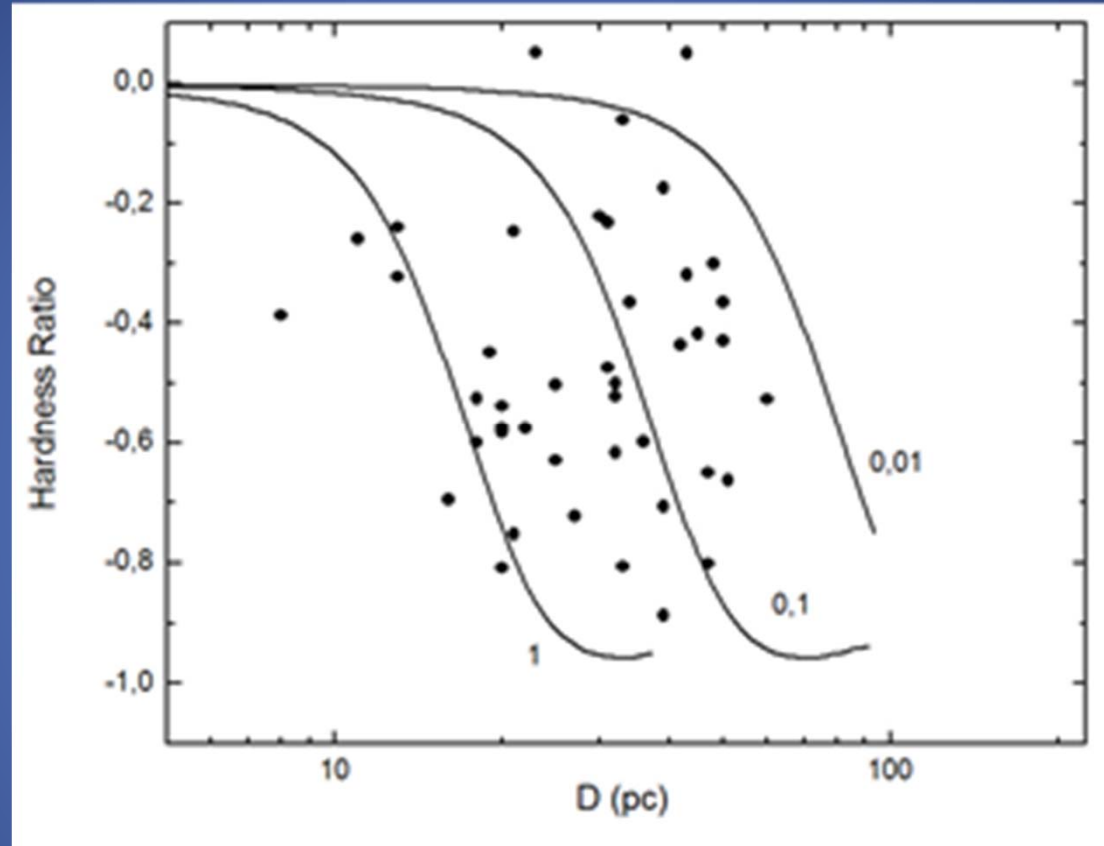
X-Ray emission of SNRs

Hardness ratio:

$$HR = \frac{\Phi(1.2; 2.6) - \Phi(0.35; 1.2)}{\Phi(0.35; 8.0)}$$

In L10 for differentiation of SNRs from other Chandra X-ray sources the criterion that $HR < 0$ is used.

X-Ray emission of SNRs

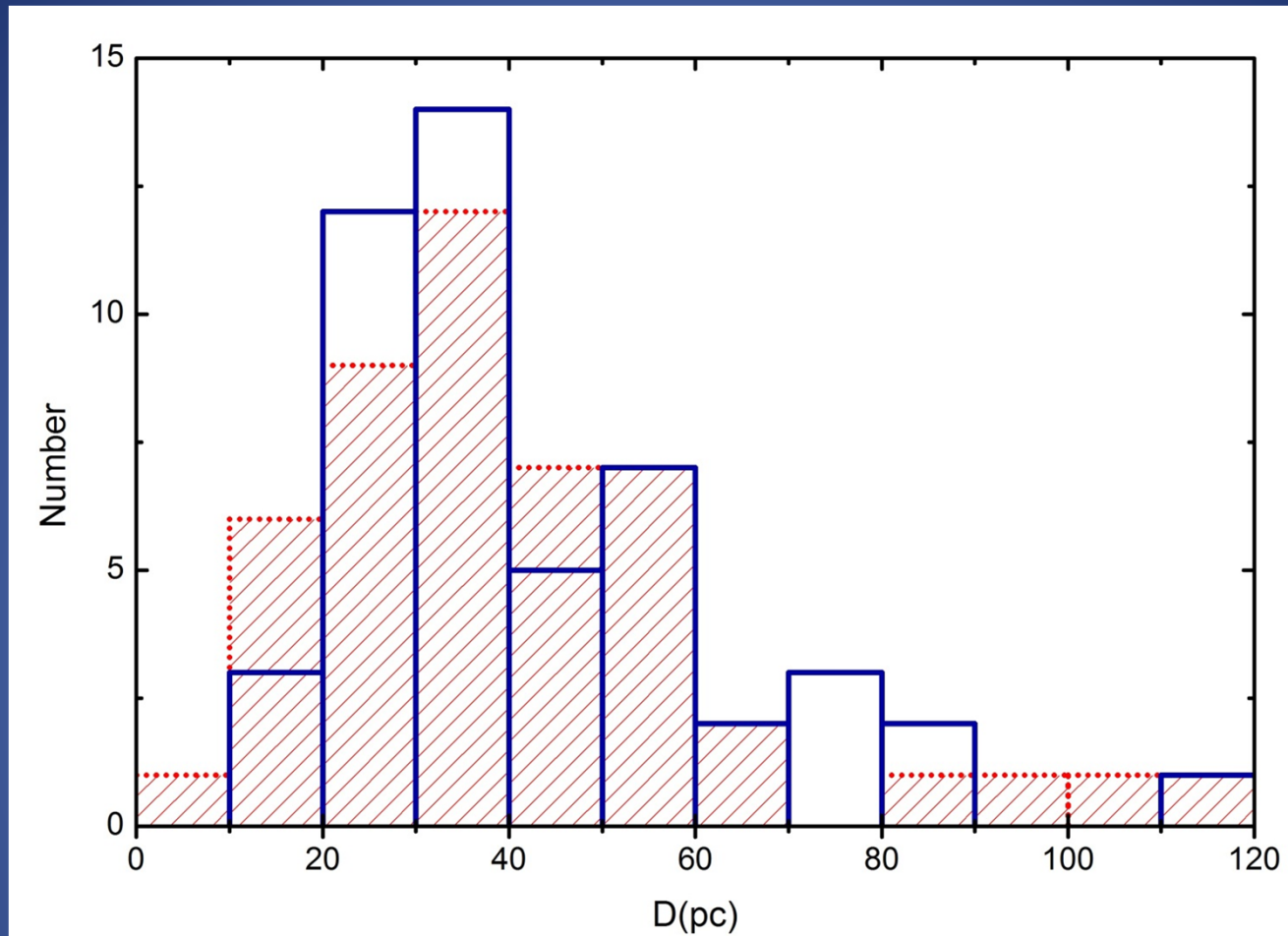


The evolution of the Hardness ratio with the diameter of SNR for $n_0=0.01$, 0.1 and 1 cm^{-3} , $E_0=10^{51} \text{ erg}$, $N_H=10^{21} \text{ cm}^{-2}$ and $d=817 \text{ kpc}$. The observational points (circles) are calculated by using the data from [T11](#).

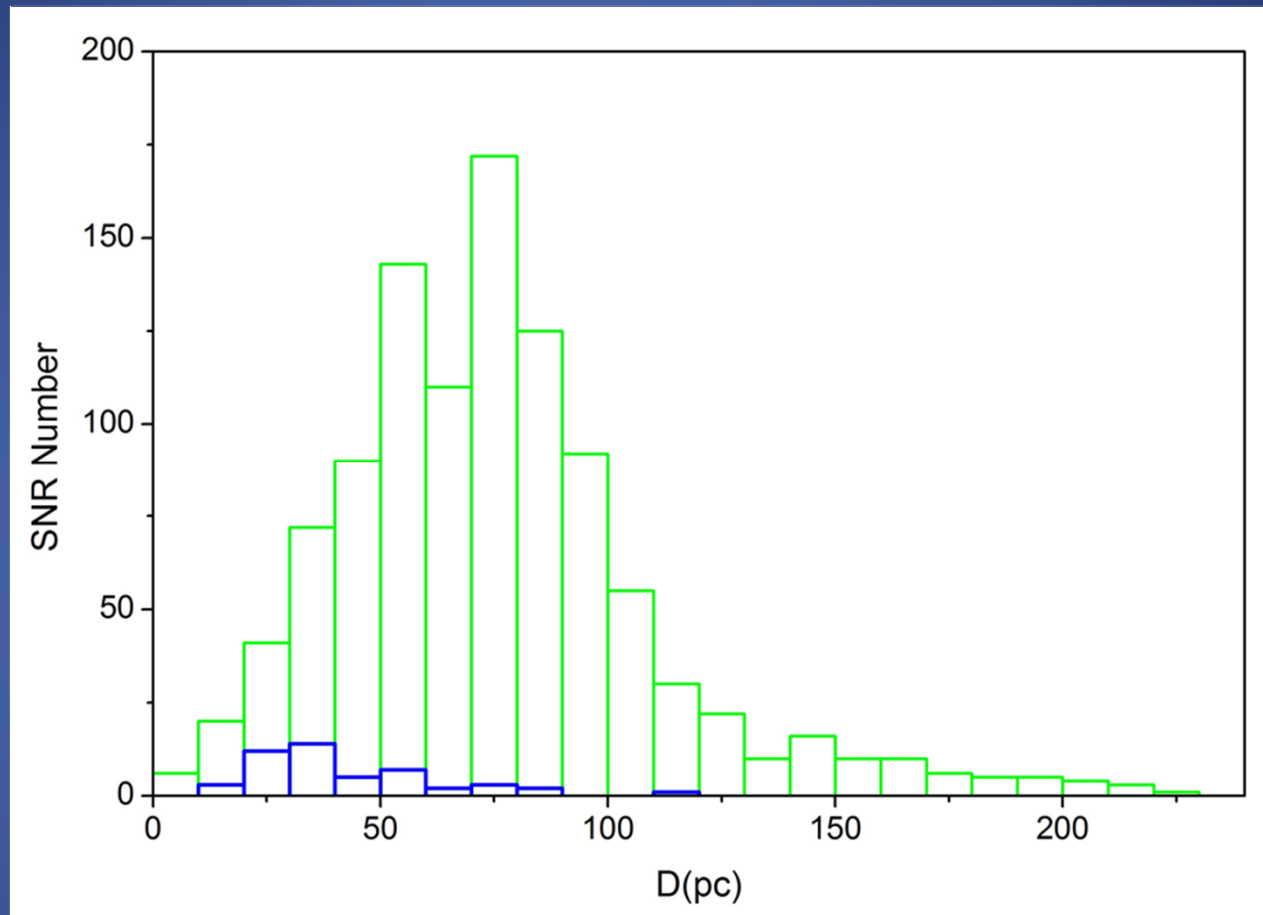
SNRs with HR around -0.5 make up statistically more complete and uniform subset of objects

For 48 of 82 SNRs in L10 $-0.7 \leq HR \leq -0.3$.

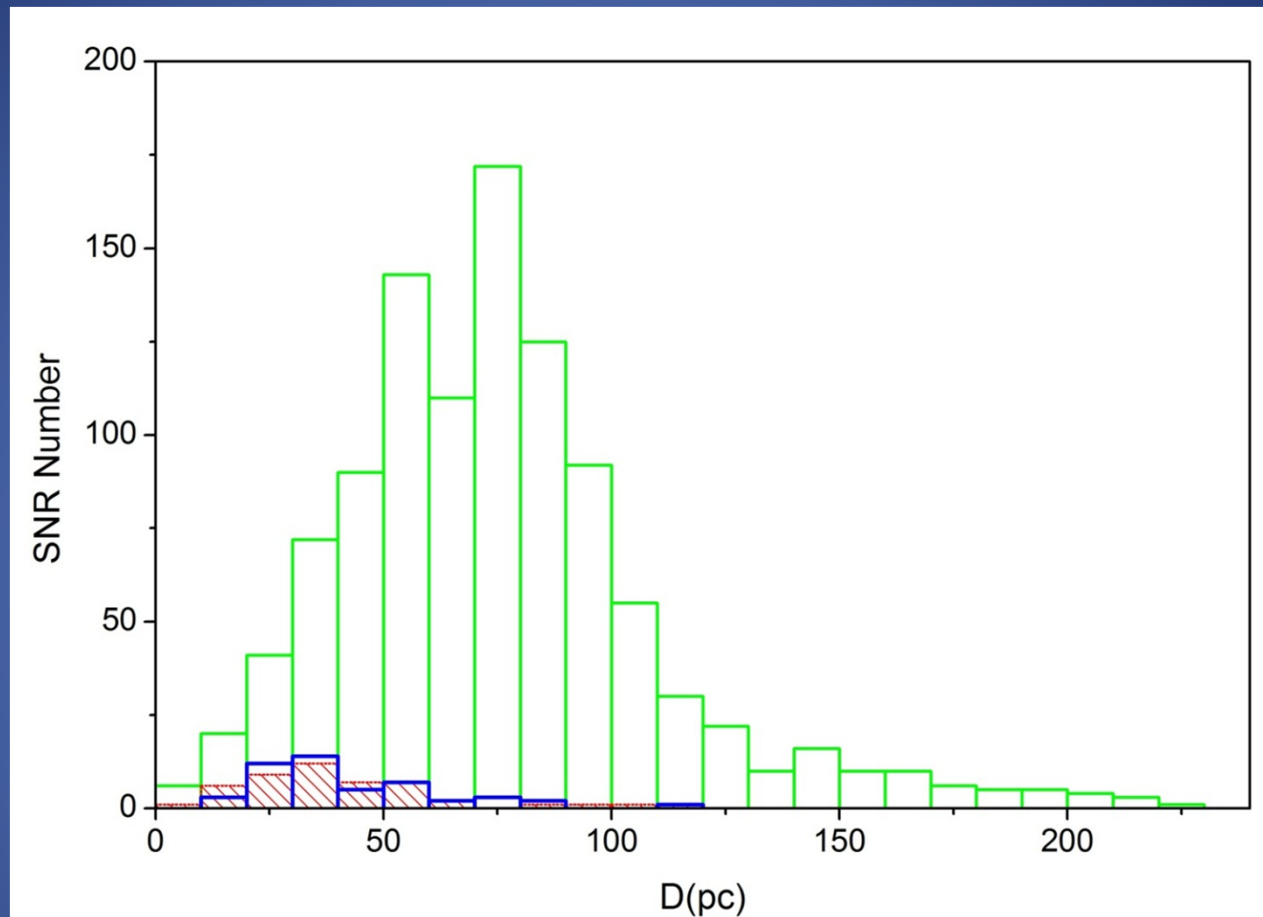
	D_{en}	D_{min}	D_{max}	$\frac{L_{\text{X,max}}}{L_{\text{X,min}}}$	$\frac{C_{\text{total,max}}}{C_{\text{total,min}}}$
	(pc)	(pc)	(pc)		
Observed SNRs					
All 137 SNRs	49.6 ± 28.8				
82 X-ray SNRs	40.3 ± 20.6	8	111	157	261
48 SNRs	39.9 ± 22.6	8	111	90	197
Modeled SNRs					
Run1	41.0 ± 20.6	16	109	165	179
Run2	39.2 ± 21.4	16.1	124	79	88
Run3	39.0 ± 19.5	12.1	104	100	380
Avrg over 20 runs	39.1 ± 18.4	13.1	107	104	210



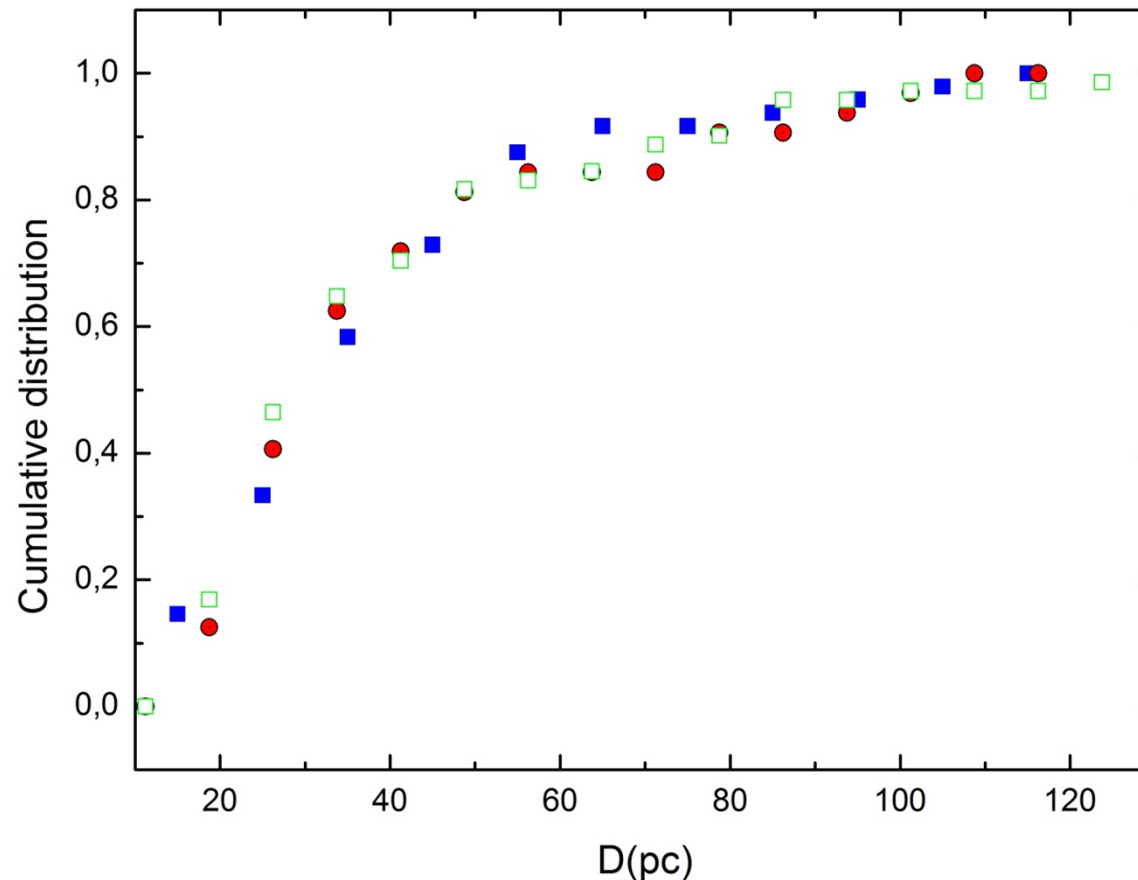
The distribution of diameters of SNRs with $-0.7 \leq HR \leq -0.3$ in M33 (doted lines) and modeled SNRs (solid lines). The data for 48 SNRs in M33 are from **L10**, the 49 modeled SNRs for the case of 10% hot gas filling-factor and birthrate 150 yr^{-1} . Size of bins is 10 pc



Distribution of 1050 modeled SNRs (green thin lines), and 49 of them with $-0.7 \leq HR \leq -0.3$ (solid lines).



Distribution of 1050 modeled SNRs (green thin lines) and of SNRs with $-0.7 \leq HR \leq -0.3$ in M33 (dotted lines) and modeled SNRs (solid lines).



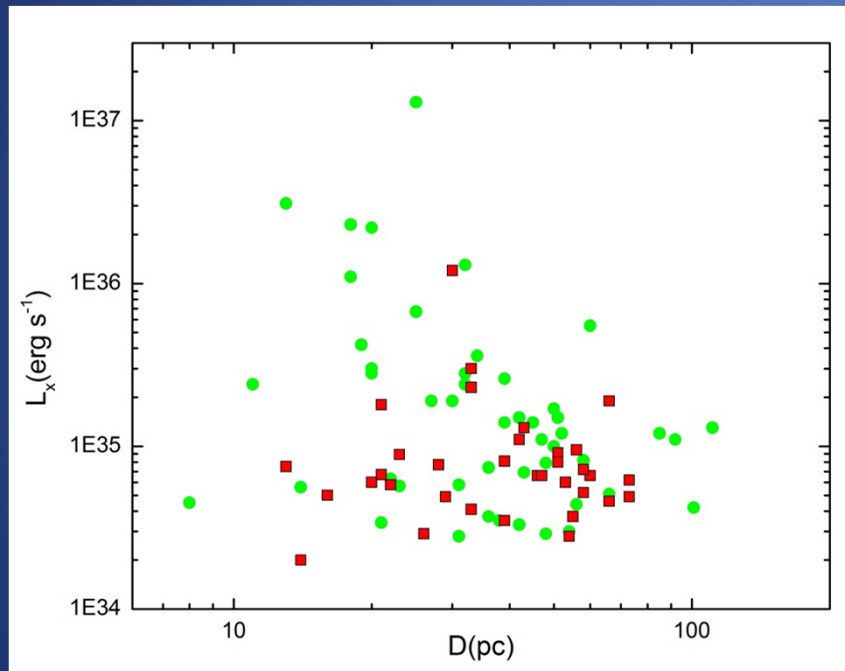
$$N(D) \sim D^\eta$$

$$\eta = 1.2 - 1.6,$$

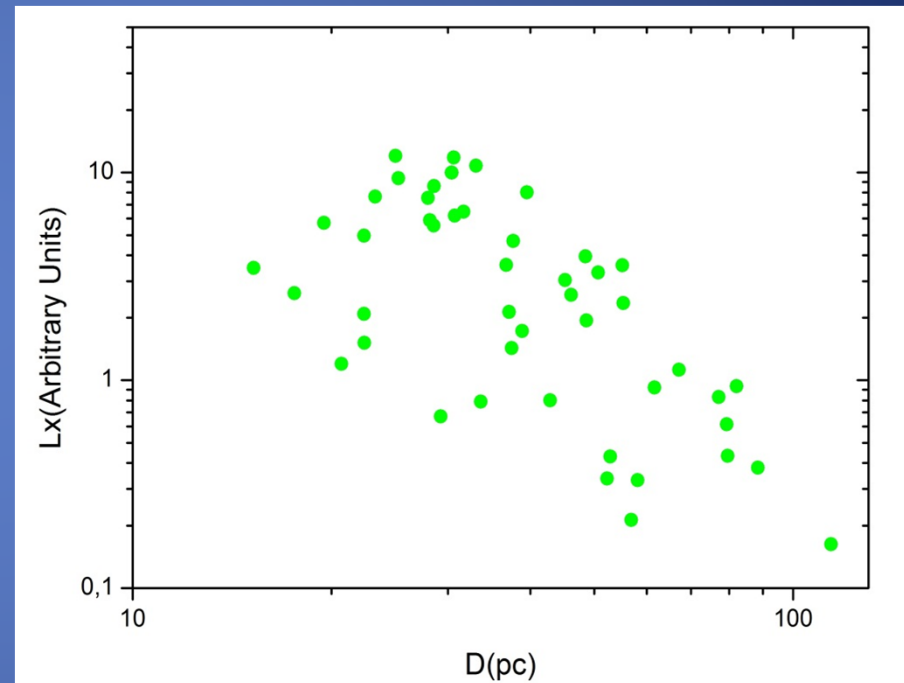
not 2.5

The normalized cumulative $N - D$ distribution for X-ray SNRs with $-0.7 \leq \text{HR} \leq -0.3$ in M33 (full squares) and two representative modeled subsets of SNRs. The data for 48 SNRs in M33 are from [L10](#), for the modeled SNRs the filling-factor of the hot phase of the ISM is 10%

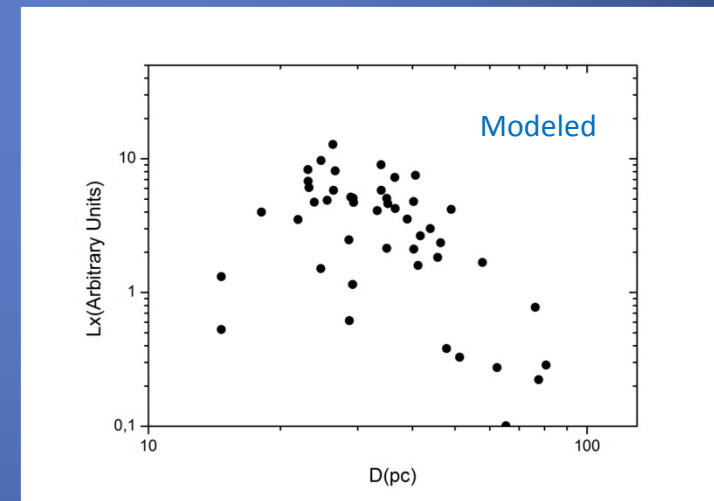
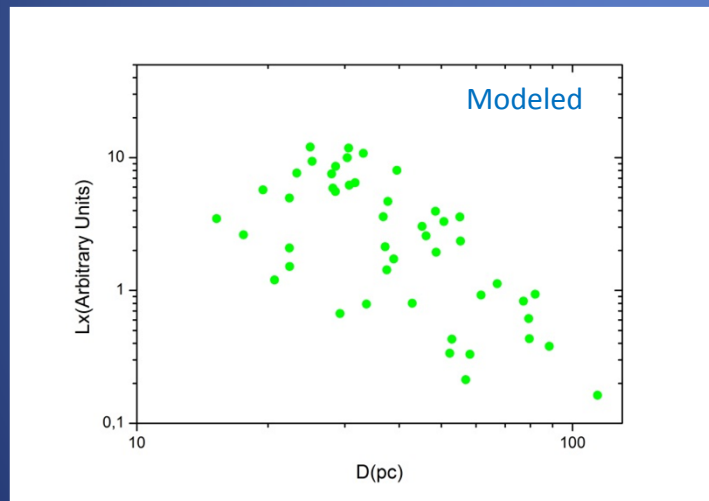
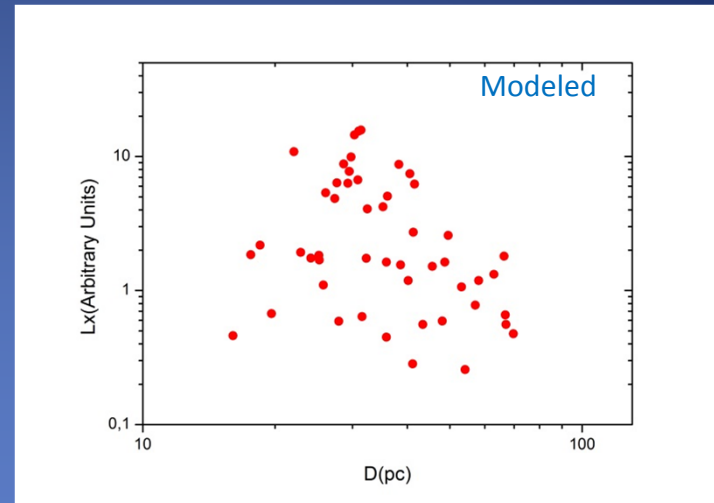
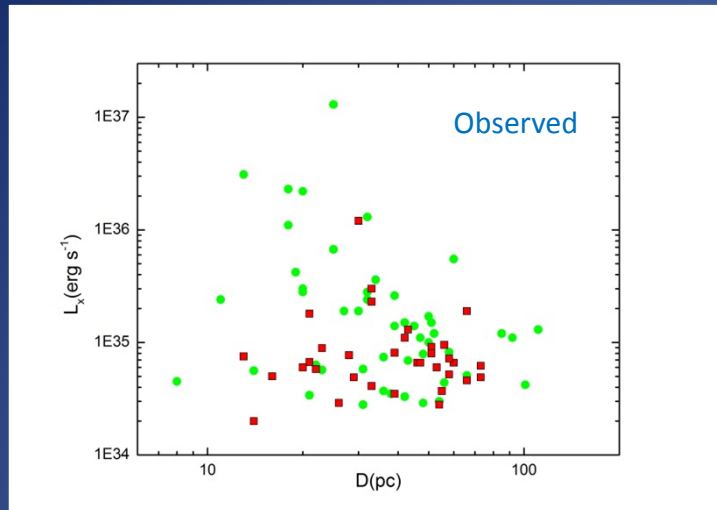
No L_X -D correlation



L_X -D dependence of observed SNRs from **L10**. 48 SNRs with $-0.7 \leq \text{HR} \leq -0.3$ are denoted with Circles, remaining 34 SNRs with squares



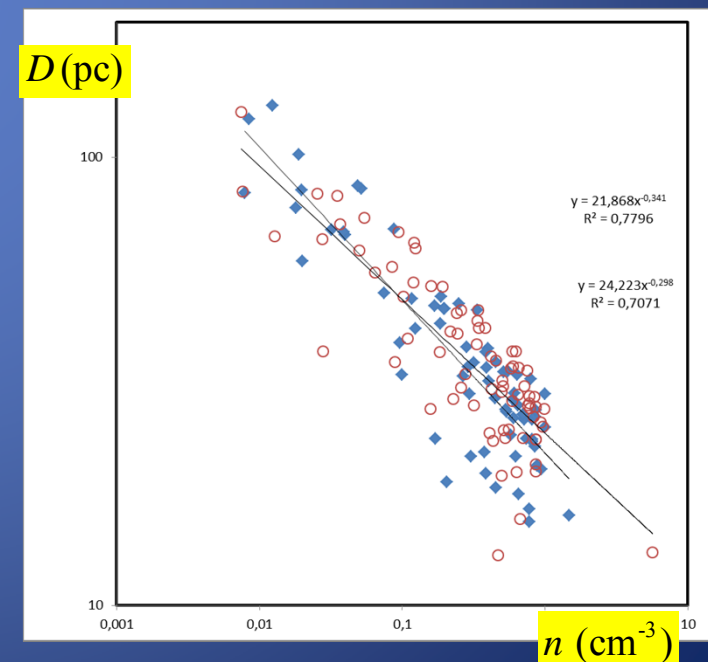
L_X -D dependence of modeled SNRs with $-0.7 \leq \text{HR} \leq -0.3$. Filling factor of hot gas 10%



$L_x - D$ dependences

$n - D$ dependence

- For the first time McKee and Ostriker (1977) for a small sample of galactic SNRs found the correlation between mean density within the remnant n and the Diameter. They used this correlation to support a multiphase model of the ISM .
- Berkhujusen (1986) for the large sample of SNRs found this dependence as $D_R \propto n^{-0.39}$ where D_R - radio diameter.
- For our modeling SNRs



M33

- Filling factor of the hot phase of the ISM $\cong 10\%$
- SNRs birthrate $130-150 \text{ yr}^{-1}$
- The number of SNRs with $M_s \geq 2$ - $1000-1200$
- SNRs occupy a volume ($h=1\text{kpc}$, $r=4.3 \text{ kpc}$) $< 1\%$,
- but $\leq 30\%$ of the projection surface.

Acknowledgments - Thanks

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THANK YOU FOR YOUR ATTENTION